Solution

EE103_Fall'18_Final Examination

Dec. 12, 2018 4:00-7:00 p.m.

Name____________________ ID______________________

I. You are allowed to use **2 pages of formulas and tables**, but not any concept descriptions, problem solutions or mathematical derivations.

II. The final course grade will be based on the following weights

Quiz Average	20% (higher 6 out of 8)
Midterm Exam	30%
Final Exam	50%

This final exam presents 6 problems

[1] 20 points ___________

[2] 15 points ___________

[3] 20 points ___________

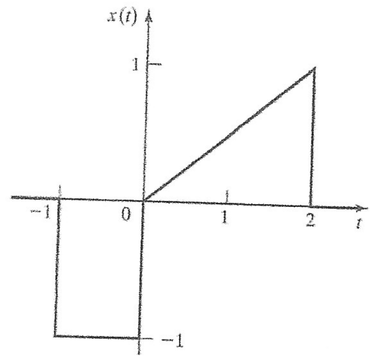
[4] 20 points ___________

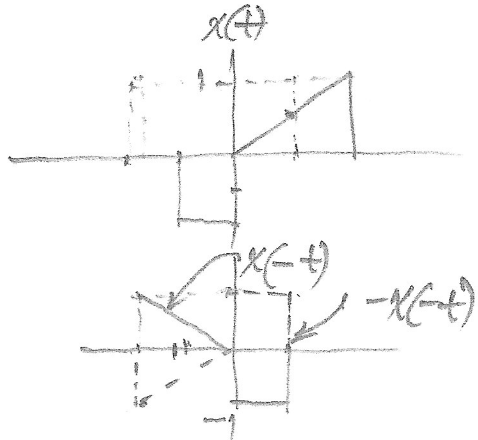
[5] 15 points ___________

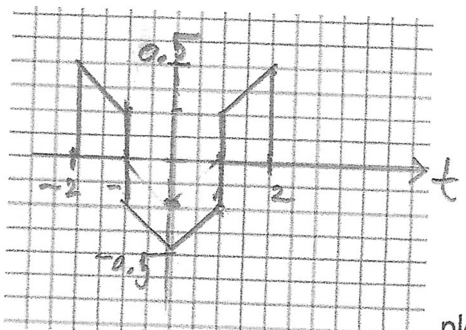
[6] 10 points ___________

Total 100 points ___________

[1] (20points) Given a graph below <u>plot $x_{even}(t)$, $x_{odd}(t)$ on the graph below with proper coordinate values.</u>

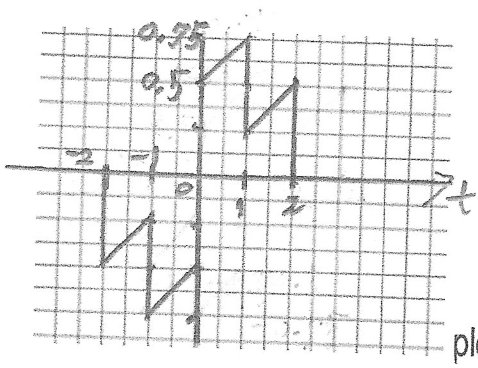






$$x_{even}(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$

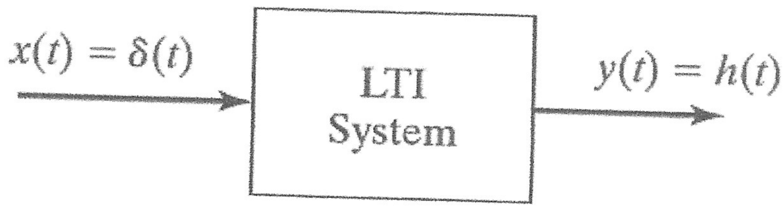
plot $x_{even}(t)$ here (10 pts)



$$x_{odd}(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$

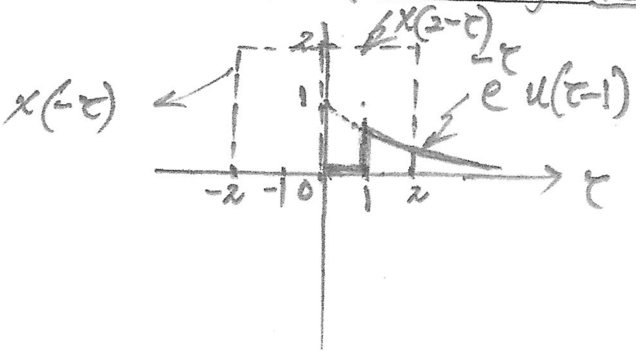
plot $x_{odd}(t)$ here (10 pts)

[2] (15 points) A linear time-invariant (LTI) system is described below.



For h(t) = $e^{-t}\,u(t-1)$, x(t) = 2 (u(t) - u(t-2)), **find y(t) value for t=2 and a mathematical description for y(t) for t >3.**

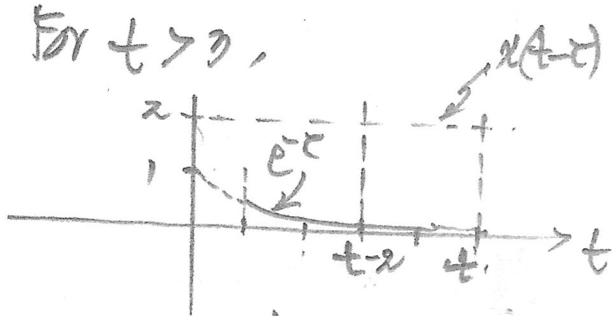
(5 pts) y(2) = $2(e^{-1} - e^{-2}) = 2e^{-1}(1 - e^{-1})$



$$y(2) = \int_1^2 2e^{-\tau}\,d\tau = 2\left[\frac{e^{-\tau}}{-1}\right]_1^2 = 2(e^{-1} - e^{-2})$$

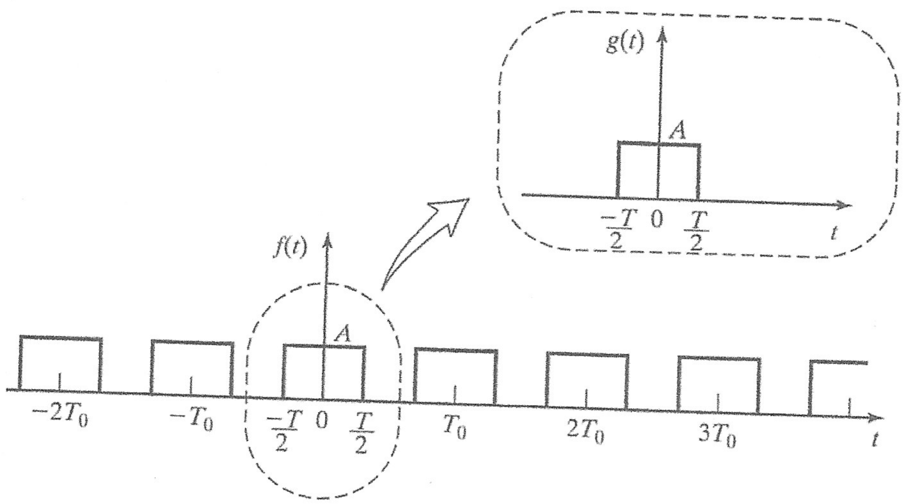
(10 pts) y(t) = $2e^{-t}(e^{2} - 1)$

For $t > 3$,



$$y(t) = 2\int_{t-2}^{t} e^{-\tau}\,d\tau = 2\left.\frac{e^{-\tau}}{-1}\right|_{t-2}^{t} = 2\left[e^{-t+2} - e^{-t}\right] = 2e^{-t}(e^{2} - 1)$$

[3] (20 points) A periodic function v(t) is plotted below.



(a) (5 pts) **Write down a mathematical description of v(t)** by using a generating function g(t) and a train of delta function $\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$

v(t) = g(t) __*__ $\delta_T(t)$. In other words, what operation would you use for ______.
Is it addition, multiplication or convolution?

convolution

(b) (15 pts) **Find Fourier transform V(ω) of v(t).**

Step 1 (5 pts) First derive G(ω) of g(t) using Fourier transform.

$$G(\omega) = \int_{-\frac{T}{2}}^{\frac{T}{2}} g(t) e^{-j\omega t} dt = A \frac{e^{-j\omega t}}{-j\omega}\Big|_{-\frac{T}{2}}^{\frac{T}{2}} = A \frac{1}{-j\omega}\left(e^{-j\omega\frac{T}{2}} - e^{-j\omega(-\frac{T}{2})}\right)$$

$$= A \frac{1}{j\omega}\left(e^{+j\omega\frac{T}{2}} - e^{-j\omega\frac{T}{2}}\right) = A \frac{1}{2j\,\omega\frac{T}{2}} \cdot T\left(e^{+j\omega\frac{T}{2}} - e^{-j\omega\frac{T}{2}}\right)$$

$$= AT \frac{\sin \omega\frac{T}{2}}{\omega\frac{T}{2}} = AT\,\mathrm{sinc}\left(\omega\frac{T}{2}\right)$$

Step 2 (10 pts) Find V(ω)

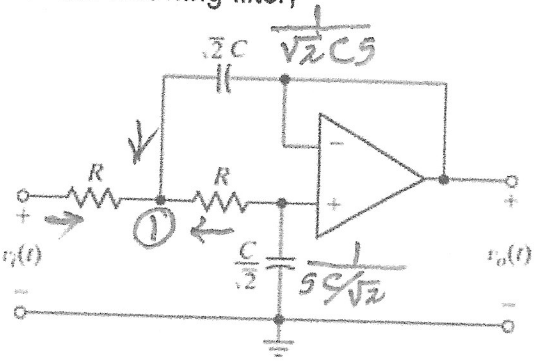
$$\Delta_T(\omega) = \mathcal{F}[\delta_T(t)] = \sum_{k=-\infty}^{\infty} \omega_0\, \delta(\omega - k\omega_0), \quad \omega_0 = \frac{2\pi}{T_0}$$

$$V(\omega) = G(\omega) \times \Delta_T(\omega)$$

$$= AT\,\mathrm{sinc}\left(\omega\frac{T}{2}\right) \cdot \sum_{k=-\infty}^{\infty} \omega_0\, \delta(\omega - k\omega_0)$$

$$= AT\omega_0 \sum_{k=-\infty}^{\infty} \mathrm{sinc}\left(k\omega_0\frac{T}{2}\right)$$

[4]. (20 points) For the following filter,



(a). (15 pts) Find H(s) = $V_o(s)/V_i(s)$

KCL at node ① : $\dfrac{V_i(s) - V_①(s)}{R} + \dfrac{V_o(s) - V_①(s)}{R} + \left(V_o(s) - V_①(s)\right)\sqrt{2}Cs = 0$

× R both sides ⇒ $V_i(s) - V_①(s) + V_o(s) - V_①(s) + \left(V_o(s) - V_①(s)\right)\sqrt{2}RCs = 0$ (1)

And $V_①(s) = V_o(s) + R\frac{C}{\sqrt{2}}s\, V_o(s) = V_o(s)\left(1 + \frac{RC}{\sqrt{2}}s\right)$ (2)

From (1) & (2)

$$V_i(s) = (2 + \sqrt{2}RCs)V_①(s) - V_o(s)(1 + \sqrt{2}RCs)$$
$$= (2 + \sqrt{2}RCs)\, V_o(s)\left(1 + \frac{RC}{\sqrt{2}}s\right) - V_o(s)(1 + \sqrt{2}RCs)$$
$$= V_o(s)\Big[\underbrace{(2 + \sqrt{2}RCs)\left(1 + \frac{RCs}{\sqrt{2}}\right)}_{= 2 + \sqrt{2}RCs + \sqrt{2}RCs + (RCs)^2} - 1 - \sqrt{2}RCs\Big]$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{(RCs)^2 + \sqrt{2}RCs + 1}$$

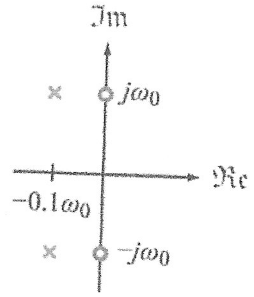
(b). (5 pts) Determine the property of this filter. Is it a low-pass or high-pass filter? Justify your answer.

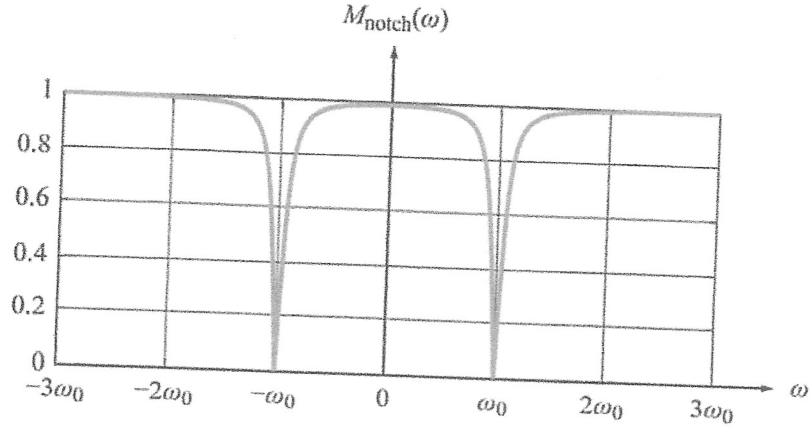
$$H(\omega) = \frac{1}{1 - (RC\omega)^2 + j\sqrt{2}RC\omega}$$

$$|H(\omega)| = \frac{1}{\sqrt{(1 - (RC\omega)^2)^2 + 2(RC\omega)^2}} \to 0 \text{ as } \omega \to \infty$$
$$= 1 \text{ at } \omega = 0$$

LP filter

[5]. (15 points) The characteristics of a filter is described below by using locations of poles and zeros (left figure) and also by a plot of its magnitude vs. ω plot (right figure).





(a). (10 pts) Derive $M_{notch}(s)$ by using the information above.

$$M_{notch}(s) = K \frac{(s+j\omega_0)(s-j\omega_0)}{(s+0.1\omega_0+j\omega_0)(s+0.1-j\omega_0)} = K\frac{s^2+\omega_0^2}{(s+0.1\omega_0)^2+\omega_0^2}$$

$$M_{notch}(\omega) = K\frac{-\omega^2+\omega_0^2}{(j\omega+0.1\omega_0)^2+\omega_0^2} \quad \text{and} \quad M_{notch}(0) = K\frac{\omega_0^2}{1.01\,\omega_0^2} = 1$$

thus $K = \frac{1}{1.01}$

$$\Rightarrow M_{notch}(s) = \frac{1}{1.01}\,\frac{s^2+\omega_0^2}{(s+0.1\omega_0)^2+\omega_0^2}$$

(b). (5 pts) Find the output y(t) for t = 0_+ for input signal x(t) = cos ($2\omega_0 t$).

$$X(s) = \frac{s}{s^2+(2\omega_0)^2}$$

$$Y(s) = M_{notch}(s)\,X(s) = \frac{1}{1.01}\,\frac{s^2+\omega_0^2}{(s+0.1\omega_0)^2+\omega_0^2}\cdot\frac{s}{s^2+(2\omega_0)^2}$$

$$y(0_+) = \lim_{s\to\infty} s\,Y(s) = \frac{1}{1.01}\lim_{s\to\infty}\frac{(s^2+\omega_0^2)\,s^2}{[(s+0.1\omega_0)^2+\omega_0^2](s^2+(2\omega_0)^2)}$$

$$= \frac{1}{1.01}$$