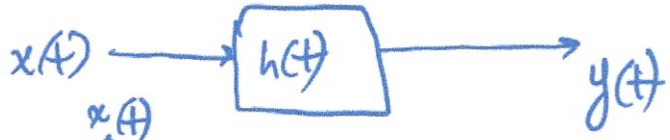
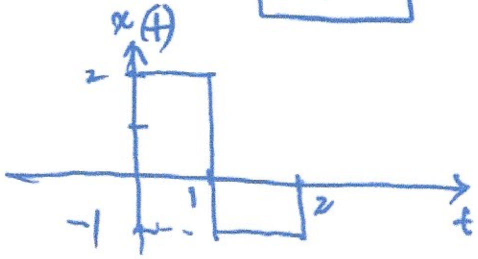
ECE 103 HW #2 Solution

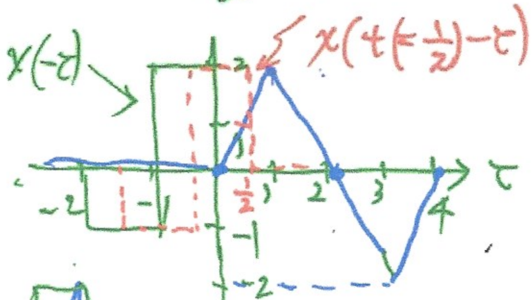
[1] 3-4 with h(t) as shown in Fig P3-4 (d)





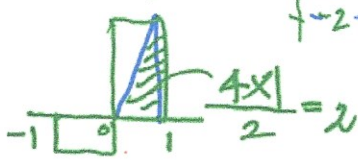
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

(d) case

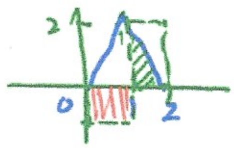


$t=0 \quad y(t) = 0$

$t=1 \quad y(t) = 2$

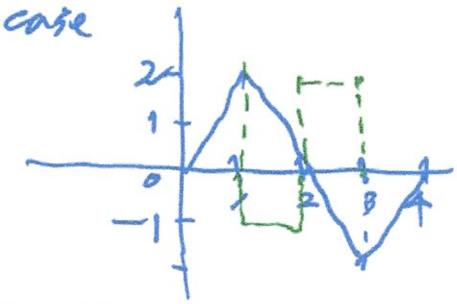


$t=2 \quad y(t) = -\frac{2\times1}{2} + \frac{4\times1}{2} = +1$



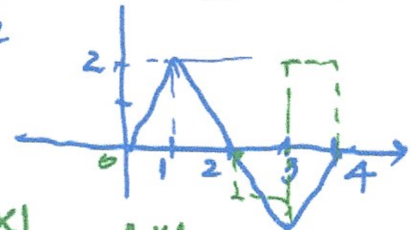
[1] (continued)

t=3 case



$$y(t) = -\frac{1\times2}{2} - \frac{4\times1}{2} = -3$$

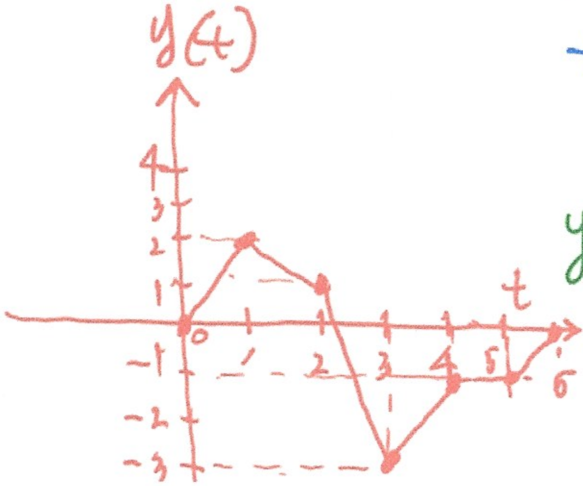
t=4 case



$$y(t) = \frac{+2\times1}{2} - \frac{4\times1}{2} = -1$$

t=5 case

$$y(t) = \frac{-2\times1}{2} + 0 = -1$$



ECE103_F18_HW#2 Solution

[2].

(3-8)

$$[f(t)*g(t)]*h(t) = \int_{-\infty}^{\infty} h(t-s)\left[\int_{-\infty}^{\infty} f(s-\tau)g(\tau)\,d\tau\right]ds$$

$$= \int_{-\infty}^{\infty} g(\tau)\left[\int_{-\infty}^{\infty} h(t-s)f(s-\tau)\,ds\right]d\tau \text{ , let } s-\tau=\sigma$$

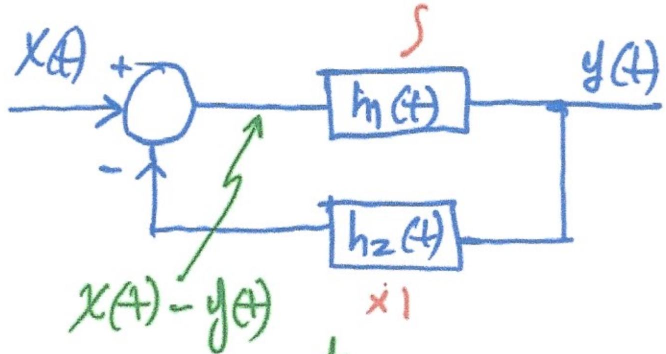
$$= \int_{-\infty}^{\infty} g(\tau)\left[\int_{-\infty}^{\infty} h(t-\tau-\sigma)f(\sigma)\,d\sigma\right]d\tau \text{ , let } s=t-\sigma, \; ds=-d\sigma$$

$$= \int_{-\infty}^{\infty} g(\tau)\left[\int_{-\infty}^{\infty} h(s-\tau)f(t-s)[-ds]\right]d\tau$$

$$= \int_{-\infty}^{\infty} f(t-s)\left[\int_{-\infty}^{\infty} h(s-\tau)g(\tau)\,d\tau\right]ds$$

$$= f(t)*[g(t)*h(t)]$$

[4] (3-18)



$h_1(t) = u(t)$

$h_2(t) = \delta(t)$

$$y(t) = \int_0^t x(\tau) - y(\tau)\, d\tau$$

$$\frac{dy(t)}{dt} = x(t) - y(t)$$

$$\Rightarrow \boxed{\frac{dy(t)}{dt} + y(t) = x(t)}$$

[5] (3.20)

$y(t) = \cos t \; x(t)$, $\quad y_1(t) = \cos t \; x_1(t)$

a) For $x(t) = a x_1(t) + b x_2(t)$ $\quad y_2(t) = \cos t \; x_2(t)$

$$y(t) = \cos t \left(a x_1(t) + b x_2(t)\right)$$
$$= a \cos t \; x_1(t) + b \cos t \; x_2(t)$$
$$= a y_1(t) + b y_2(t)$$ thus linear (✓)

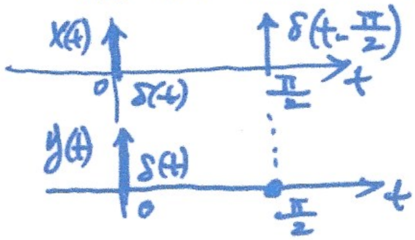
b) time invariant?

$\cos t$ is time-varying! $\Rightarrow$ <u>not</u> time invariant

c) $x(t) = \delta(t)$, $y(t) = \cos t \; \delta(t) = 1 \cdot \delta(t)$

d) $x(t) = \delta(t - \pi/2)$, $y(t) = \cos t \; \delta(t - \frac{\pi}{2}) = \cos \frac{\pi}{2} \cdot \delta(t - \frac{\pi}{2}) = 0$

This not time invariant



[6]

(i) Characteristic equation: $s+3=0$, solution $s=-3$

$\implies y_c(t) = Ce^{-3t}u(t)$

Forced response of the form: $y_p(t) = Pu(t)$ where $\frac{dy_p(t)}{dt}+3y_p(t) = 3u(t)$

$\implies 0+3Pu(t) = 3u(t) \implies P=1$

$y(t) = y_c(t)+y_p(t) = (Ce^{-3t}+1)u(t)$

Need $y(0) = C+1 = -1 \implies C=-2$

$\implies y(t) = (-2e^{-3t}+1)u(t)$

This clearly satisfies the differential equation and initial conditions because

$\frac{dy(t)}{dt}+3y(t) = 6e^{-3t}+3(-2e^{-3t}+1) = 3. t>0$

$y(0) = -2e^{-3.0}+1 = -1$

(ii) Characteristic equation: $s+3=0$, solution $s=-3$

$\implies y_c(t) = Ce^{-3t}u(t)$

Forced response of the form $y_p(t) = Pe^{-2t}u(t)$ where $\frac{dy_p(t)}{dt}+3y_p(t) = 3e^{-2t}u(t)$

$\implies (-2P+3P)e^{-2t}u(t) = 3e^{-2t}u(t) \implies P=3$

$y(t) = y_c(t)+y_p(t) = (Ce^{-3t}+3e^{-2t})u(t)$

Need $y(0) = C+3 = 2 \implies C=-1$

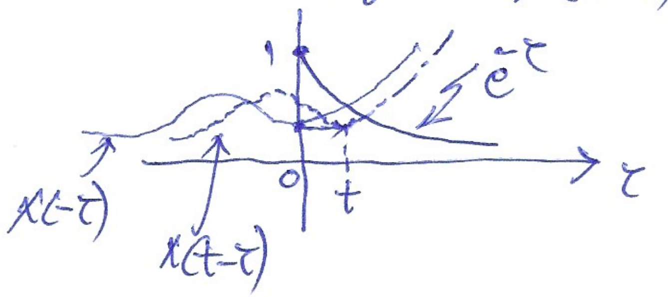
$\implies y(t) = (3e^{-2t}-e^{-3t})u(t)$

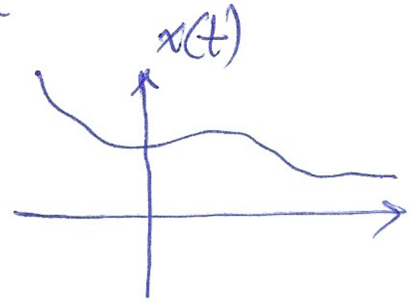
This clearly satisfies the differential equation and initial conditions since

$\frac{dy(t)}{dt}+3y(t) = (-6e^{-2t}+3e^{-3t})+3(3e^{-2t}-e^{-3t}) = 3e^{-2t}, t>0$

$y(0) = 3e^{-2.0}-e^{-3.0} = 2$

[7] $y(t) = \int_0^{\infty} e^{-\tau} x(t-\tau)\, d\tau$





a) Find $y(t) = h(t)$ for $x(t) = \delta(t)$

$$h(t) = \int_0^{\infty} e^{-\tau} \delta(t-\tau)\, d\tau = \int_0^{\infty} e^{-t} \delta(t-\tau)\, d\tau$$

$$= e^{-t} \underbrace{\int_0^{\infty} \delta(t-\tau)\, d\tau}_{=1} = e^{-t} u(t)$$

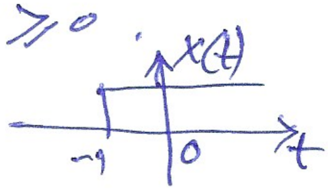
b) Is this system causal?

$h(t) = 0$ for $t < \tau$ when $0 \le \tau < \infty$

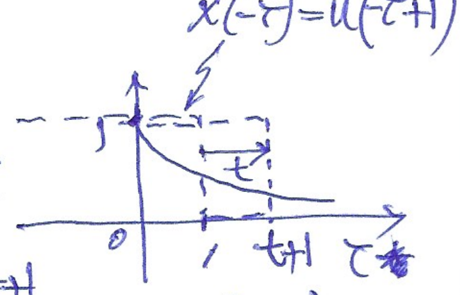
i.e zero for $t < 0$

$h(t) \neq 0$ only for $t \geq 0$.

c) $x(t) = u(t+1)$

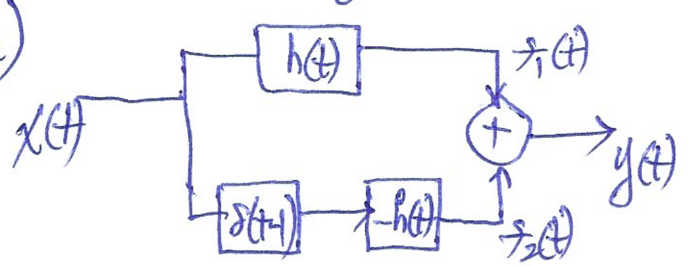


$$y(t) = x(t) * h(t)$$
$$= u(t+1) * e^{-t} u(t)$$
$$= \int_0^{\infty} e^{-\tau} u(\underbrace{t-\tau+1}_{-(\tau-1-t)})\, d\tau$$



$$= \int_0^{t+1} e^{-\tau}\, d\tau = -e^{-\tau}\Big|_0^{t+1} = 1 - e^{-(t+1)}$$

d)



For $x(t) = \delta(t)$,

$f_1(t) = \delta(t)$

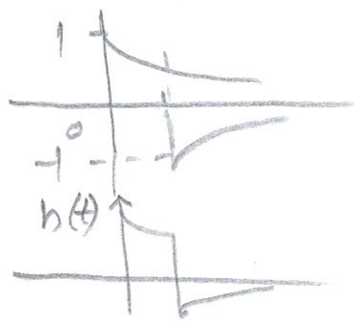
$f_2(t) = -\delta(t-1)$

$y(t) = \delta(t) - \delta(t-1)$

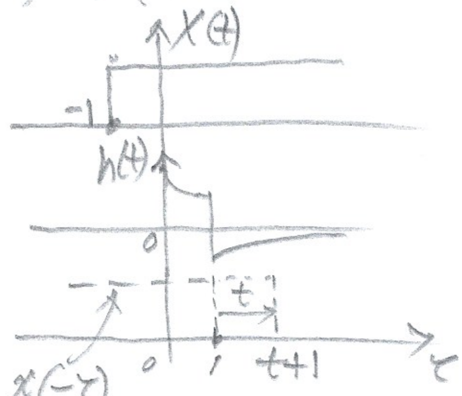
(e)(i) $x(t) = u(t+1)$

For $x(t) = u(t)$, $y(t) = 1 - e^{-t}$

ii) For $h(t) = e^{-t} - e^{-(t-1)}$



$y(t) = x(t) * h(t)$



For $t < -1$, $y(t) = 0$

$t = 0$ $\int_0^1 e^{-\tau} d\tau = -e^{-\tau}\Big|_0^1 = 1 - e^{-1} = 0.63$

For $t > 0$

$$y(t) = \int_1^{t+1} \left(e^{-\tau} - e^{-(\tau-1)}\right) d\tau + (1 - e^{-1})$$

$$= \left[-e^{-\tau} + e^{-\tau} \cdot e^{1}\right]_1^{(t+1)} + 1 - e^{-1}$$

$$= \cancel{e^{-1}} - e^{-(t+1)} + e^{+1}\left(e^{-(t+1)} - e^{-1}\right) + 1 - \cancel{e^{-1}}$$

$$= \underbrace{(e-1)}_{1.72} e^{-(t+1)}$$

$= 0.23$, $t = 1$

0.085, $t = 2$