

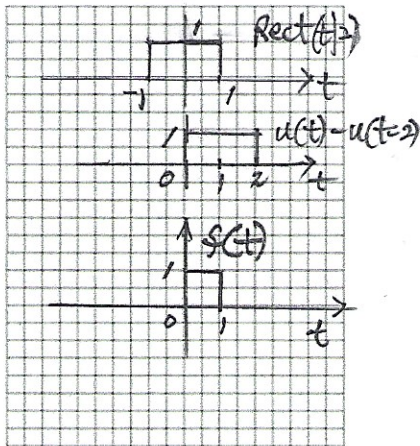
Solution

ECE103 Fall 2018 Quiz 4, Oct. 29, 2018

Name _____ Student ID No. _____

Let $f(t) = \text{Rect}(t/2) [u(t) - u(t-2)]$, where $\text{Rect}(t/2) = 1$ for $-1 < t < 1$, and 0 elsewhere.
 $u(t) = 1$ for $t > 0$, 0 for $t < 0$.

(a) (4 pts) Draw $f(t)$ in the graph below



(b) (6 pts) Find $F(\omega)$ by using the definition of Fourier Transform. (Hint: your answer can be simplified by using the "sinc $x = (\sin x)/x$ " identity. Then your final answer can be expressed as

$$F(\omega) = \left(e^{-j\frac{\omega}{2}} \right) \text{sinc} \left[\frac{\omega}{2} \right]$$

Please show your derivation process.

$$\begin{aligned} F(\omega) &= \int_0^1 1 e^{-j\omega t} dt = \left. \frac{e^{-j\omega t}}{-j\omega} \right|_0^1 = \frac{e^{-j\omega} - 1}{-j\omega} = \frac{1 - e^{-j\omega}}{j\omega} \\ &= \frac{e^{-j\frac{\omega}{2}} \left(e^{+j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}} \right)}{j\omega} = \frac{e^{-j\frac{\omega}{2}} \left(e^{+j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}} \right)}{j\omega} \\ &= \frac{e^{-j\frac{\omega}{2}} \left(\frac{\sin(\frac{\omega}{2})}{\frac{\omega}{2}} \right)}{\frac{\omega}{2}} = \frac{e^{-j\frac{\omega}{2}} \text{sinc}\left(\frac{\omega}{2}\right)}{1} \end{aligned}$$