ECE 103 Lecture 11, Oct 22, 2013

Let's review on systems concept.

Input \( \sum_{k=-\infty}^{\infty} x(t) e^{-j2\pi f_0 t} \) to system \( H(f) = \frac{1}{2} \delta(f - f_0) + \frac{1}{2} \delta(f + f_0) \) yields

\[ y(t) = \frac{1}{2} x(t) e^{-j2\pi f_0 t} + \frac{1}{2} x(t) e^{j2\pi f_0 t} \]

When \( x(t) \) is periodic with period \( T_0 \), its fundamental frequency is \( f_0 = \frac{1}{T_0} \) and angular frequency is \( \omega_0 = 2\pi f_0 \). 

\[ x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi k\omega_0 t} = C + \frac{1}{2\pi} \sum_{K=0}^{\infty} (c_k + c_k) e^{j2\pi k\omega_0 t} \]

\[ e^{j2\pi \omega_0 t} = 1 \]

\[ c_k = 1/2 \] for \( k = 0 \)

\[ c_k = 0 \] for \( k \neq 0 \)

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TABLE 4.8 Amplitude and Time Transformations

<table>
<thead>
<tr>
<th>Equation Title</th>
<th>Equation Number</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential form of Fourier series</td>
<td>(4.11)</td>
<td>( y(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi ft} )</td>
</tr>
<tr>
<td>Combined trigonometric form of Fourier series</td>
<td>(4.12)</td>
<td>( y(t) = \sum_{k=-\infty}^{\infty} A_k \cos(k \omega_0 t) + B_k \sin(k \omega_0 t) )</td>
</tr>
<tr>
<td>Trigonometric form of Fourier series</td>
<td>(4.13)</td>
<td>( y(t) = \sum_{k=-\infty}^{\infty} A_k \cos(k \omega_0 t) + B_k \sin(k \omega_0 t) )</td>
</tr>
<tr>
<td>Relation of different forms of Fourier coefficients</td>
<td>(4.14)</td>
<td>( y(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi ft} = \sum_{k=-\infty}^{\infty} A_k \cos(k \omega_0 t) + B_k \sin(k \omega_0 t) )</td>
</tr>
<tr>
<td>Fourier series coefficients formula</td>
<td>(4.15)</td>
<td>( c_k = \frac{1}{2 \pi} \int_{-\pi}^{\pi} y(t) e^{-j2\pi ft} dt )</td>
</tr>
<tr>
<td>Unit function</td>
<td>(4.16)</td>
<td>( y(t) = \frac{1}{2 \pi} )</td>
</tr>
<tr>
<td>Steady-state output expressed as Fourier series</td>
<td>(4.17)</td>
<td>( x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi ft} )</td>
</tr>
<tr>
<td>Fourier coefficients of output signal</td>
<td>(4.18)</td>
<td>( x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi ft} )</td>
</tr>
</tbody>
</table>

\[ c_k = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) e^{-j2\pi ft} dt \]

For \( k = 0 \)

\[ c_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) dt \]

For \( k \neq 0 \)

\[ c_k = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) e^{-j2\pi ft} dt \]
\[ \frac{c_k}{c_{k-1}} = \frac{k}{(k-1)} \]

\[ c_k = j \frac{k}{\pi} \]

\[ \gamma_k = \tan^{-1} \frac{k}{2\pi} \]

\[ \theta_k = 90 - \gamma_k \]

\[ \theta_k = 90 - 43.5^\circ \]

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\[ \theta_k = 90 - 90^\circ \]