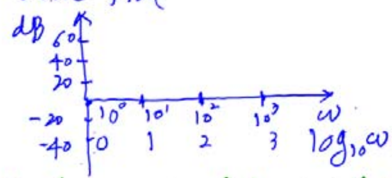
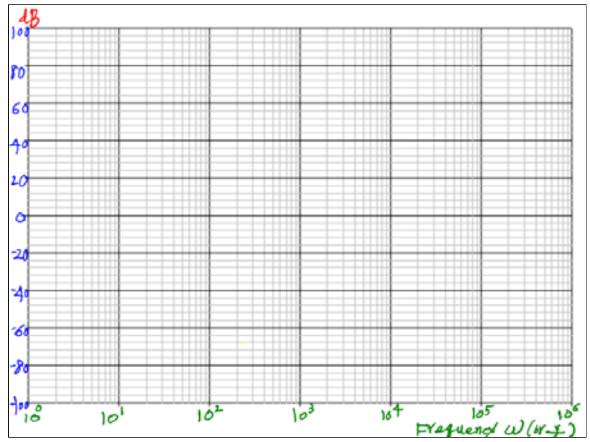
ECE 103 F18 Lecture 15 Nov. 2, 2018

Bode plot



Recall $\log_{10} A\cdot B = \log_{10} A + \log_{10} B$

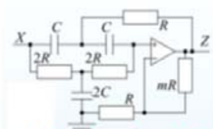
$\log_{10} \frac{A}{B} = \log_{10} A - \log_{10} B$



ECE 103L #6 Filter Design

1. In the lab we analyzed filtering 60 Hz power-line noise from ECG signal using a digital (signal processing) filter. Now let's try to an analog (circuit) filter approach to remove the 60 Hz line-noise. Following is an active twin-T notch filter with transfer function:

$$H(\omega) = \frac{Z(\omega)}{X(\omega)} = \frac{(1+m)\left((2j\omega RC)^2+1\right)}{(2j\omega RC)^2 + 4(1-m)j\omega RC + 1}$$



Here m is the ratio of the two feedback resistance which determines the gain and quality for the filter. The "drop frequency" of this twin-T notch filter is f_{drop}=1/4πRC. For designing a 60 Hz drop filter, let's use R=10 kΩ and C=133 nF.

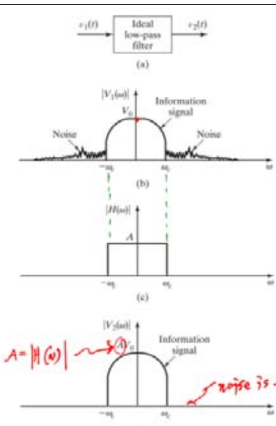
$H(\omega)=0$ when $(2j\omega RC)^2=-1$

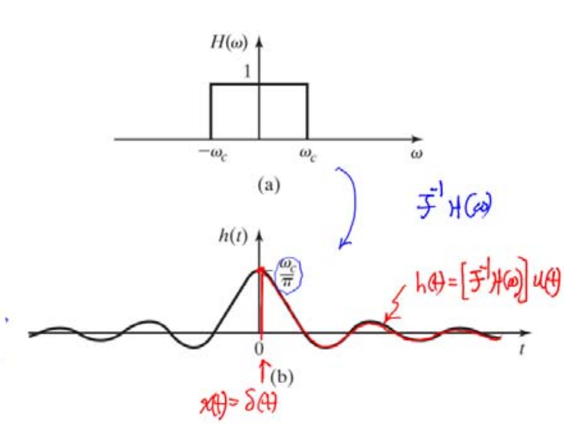
$(2\omega RC)^2=1 \Rightarrow \omega=\frac{1}{2RC} \Rightarrow \boxed{f=\frac{1}{4\pi RC}}$

(a) For m={0.8, 0.9} plot the magnitude and phase response of H(ω) with a range of f=ω/2π=[0, 200 Hz].

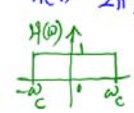
$\frac{1}{4\pi RC} = \frac{1}{4\pi\, 10^4 \times 133\times10^{-9}} = \frac{10^5}{1671.33} = 59.83$

(b) Consider the ECG signal used during the class (ecg_signal.mat) as the input (x(t)=ecg) of a 60 Hz twin-T notch filter with m=0.9. Using the functions fft() and ifft(), determine the X(ω), Z(ω), and z(t) [z(t) is the output signal from the twin-T notch filter]. Plot x(t), X(f), Z(f), and z(t) in a 4x1 subplot for the range of $-250 \le f \le 250$ and $0 \le t \le 2.5$.





$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\, e^{-j\omega t}\, d\omega$$



$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1\, e^{-j\omega t}\, d\omega = \frac{1}{2\pi}\left[\frac{e^{-j\omega t}}{-jt}\right]_{-\omega_c}^{+\omega_c}$$

$$= \frac{1}{2\pi}\frac{1}{-jt}\left[e^{-j\omega_c t} - e^{-j(-\omega_c)t}\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{+j\omega_c t} - e^{-j\omega_c t}}{jt}\right]$$

$$= \frac{1}{\pi}\left[\underbrace{\frac{e^{+j\omega_c t} - e^{-j\omega_c t}}{2j}}_{=\sin \omega_c t}\frac{\omega_c}{\omega_c t}\right]$$

$$= \frac{\omega_c}{\pi}\, \mathrm{sinc}\, \omega_c t$$

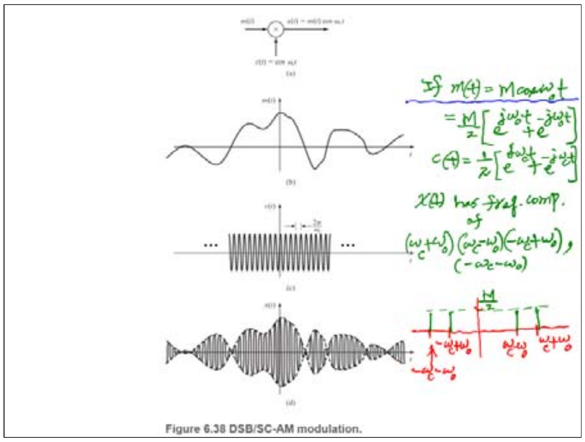


Figure 6.38 DSB/SC-AM modulation.

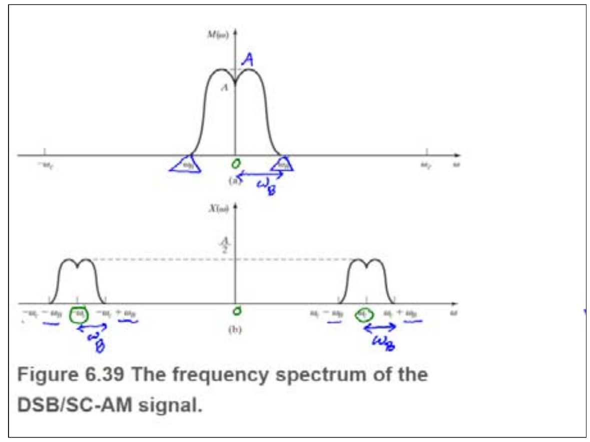
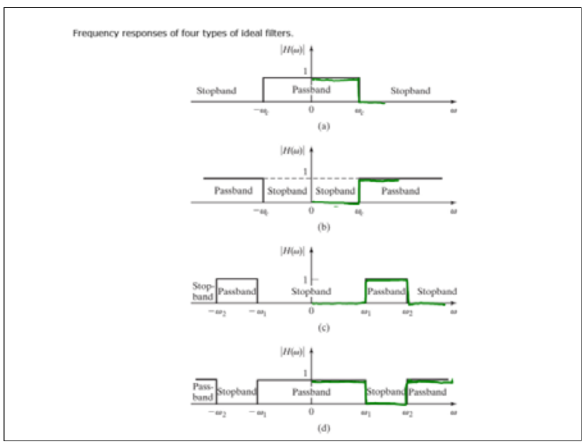
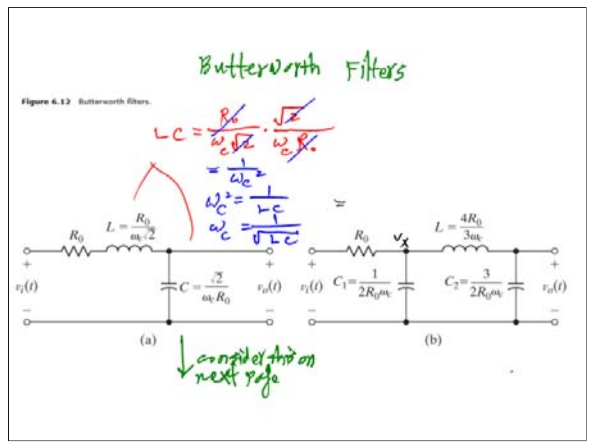
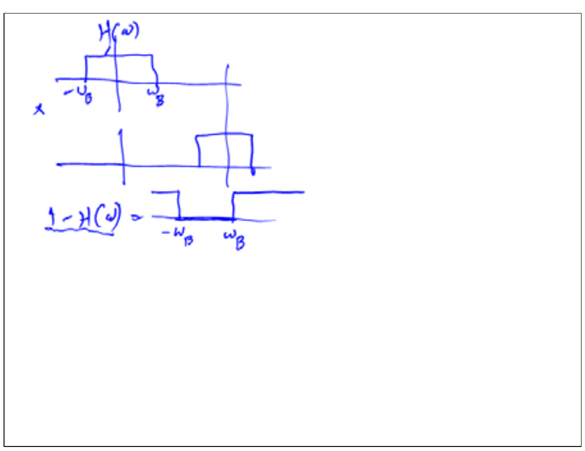


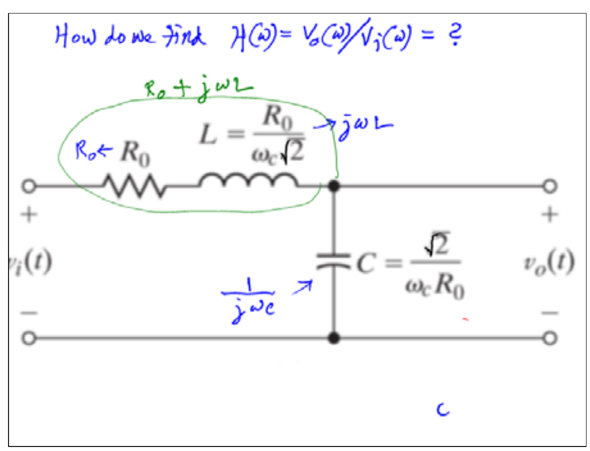
Figure 6.39 The frequency spectrum of the DSB/SC-AM signal.

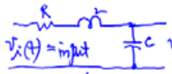
Frequency responses of four types of ideal filters.











$v_i(t)$ = input $\quad$ $v_o(t)$ = output

$L = R\frac{1}{\omega_c\sqrt{2}}$

$C = \frac{\sqrt{2}}{\omega_c R}$

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{\frac{1}{j\omega C}}{(R+j\omega L)+\frac{1}{j\omega C}}$$

$$V_o(\omega) = \frac{V_i(\omega)}{(R+j\omega L)+\frac{1}{j\omega C}} \cdot \frac{1}{j\omega C}$$

$$\frac{V_o(\omega)}{V_i(\omega)} = \left[\frac{\frac{1}{j\omega C}}{(R+j\omega L)+\frac{1}{j\omega C}}\right] = \frac{1}{1+j\omega C(R+j\omega L)}$$

$\times(j\omega C)$

$$= \frac{1}{1+j\omega RC+j\omega C(j\omega L)}$$

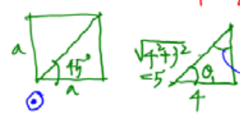
$$|H(\omega)| \quad \angle H(\omega) \qquad = \frac{1}{(1-\omega^2 LC)+j\omega RC}$$

$$\left|\frac{1}{a+jb}\right| = \frac{1}{\sqrt{a^2+b^2}}$$

$$\angle\frac{1}{a+jb} = \underset{0}{\angle 1} - \angle a+jb \qquad \frac{e^{j\theta_1}}{e^{j\theta_2}} = e^{j(\theta_1-\theta_2)}$$

$\angle a+jb$

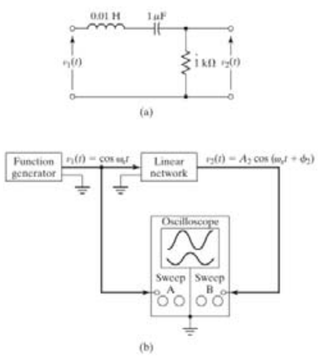
$\theta_2 = \tan^{-1}\left(\frac{b}{a}\right)$



$\theta_1 = \tan^{-1}\left(\frac{3}{4}\right) = 36.9°$

$53.1° = \tan^{-1}\frac{4}{3}$

$$|H(\omega_x)| = \frac{|V_2(\omega_x)|}{|V_1(\omega_x)|} = A_2.$$



$$H(\omega) = \frac{R}{R+j\omega L+1/j\omega C} = \frac{j(R/L)\omega}{-\omega^2+1/LC+j(R/L)\omega}$$

$$= \frac{j10^4\omega}{-\omega^2+10^8+j10^4\omega}$$

$$= \frac{1}{1+j(\omega^2-10^8)/10^4\omega}.$$

The student wrote the transfer function in polar form as

$$H(\omega) = |H(\omega)|\, e^{j\phi(\omega)}$$

where

mag.

$$|H(\omega)| = \frac{1}{\left[1+\left[(\omega^2-10^8)/10^4\omega\right]^2\right]^{1/2}}$$

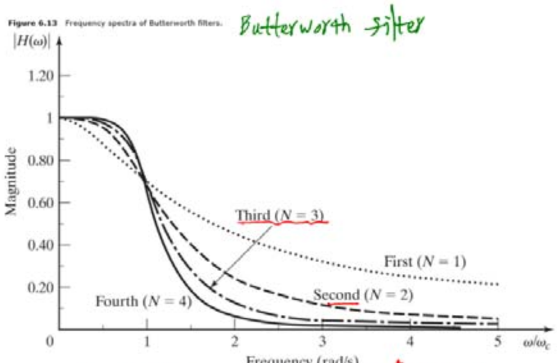
and

phase

$$\phi(\omega) = -\tan^{-1}\left[(\omega^2-10^8)/10^4\omega\right].$$

Figure 6.13 Frequency spectra of Butterworth filters.

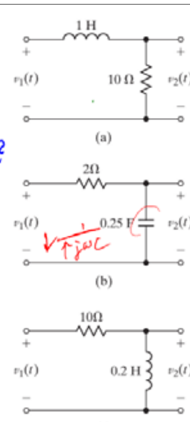
Butterworth filter



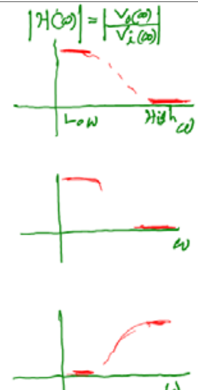
As order N increases $|H(\omega)| \rightarrow$ N=4

what happens when the freq. of $v_i(t)$ changes?

$\left|\frac{V_2(\omega)}{V_1(\omega)}\right|$ vs. ω ?



$|H(\omega)| = \left|\frac{V_o(\omega)}{V_i(\omega)}\right|$



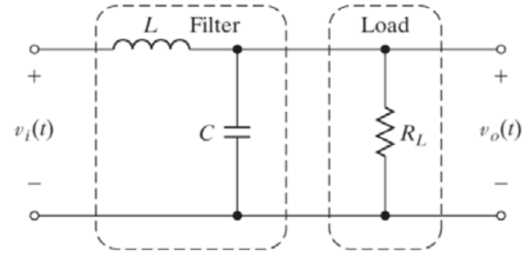
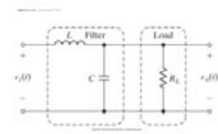


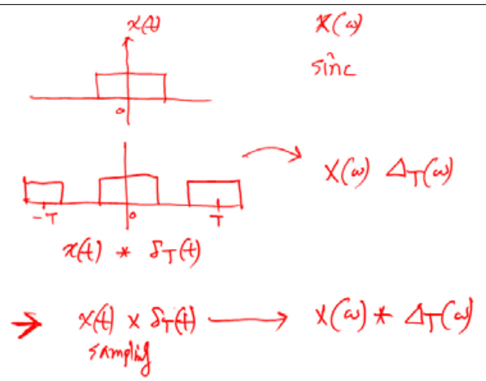
Figure 6.14 A practical filter.

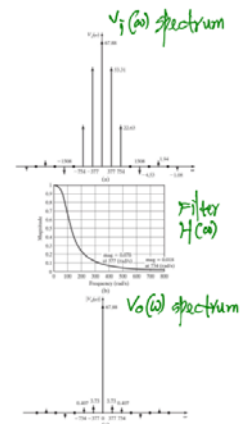
$\omega_0 = 2\pi f_0 = 2\pi(60) = 377$

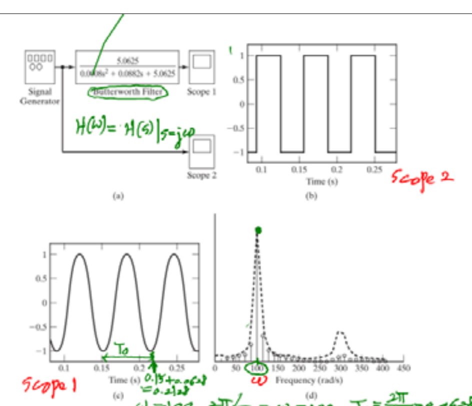
$V_1(\omega) = 53.31 \sum_{n=-\infty}^{\infty} \cancel{n=-\infty\ \infty}$ sinc(nπ / 2)[δ(ω−(n+1)377)+δ(ω−(n−1)377)].

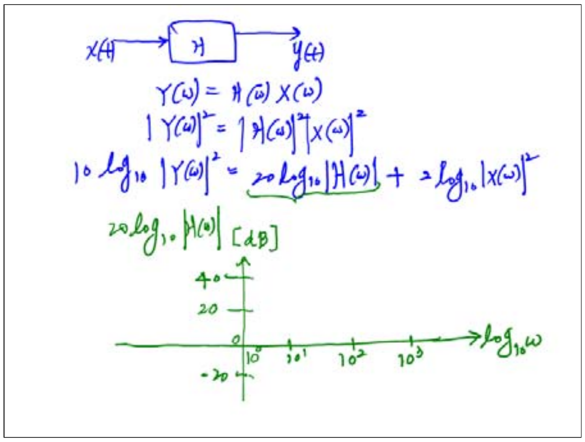
How do we find $H(\omega) = V_o(\omega)/V_i(\omega)$ for the filter below?



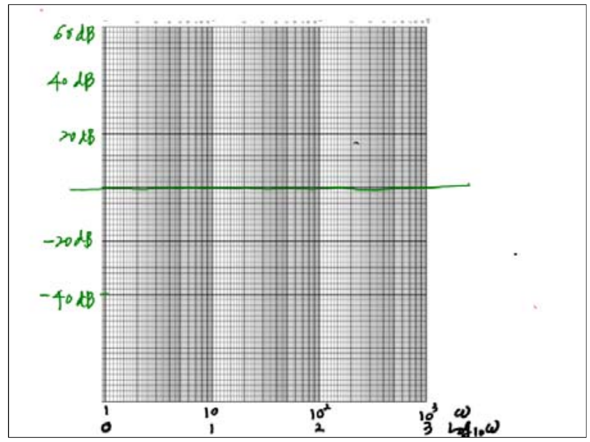




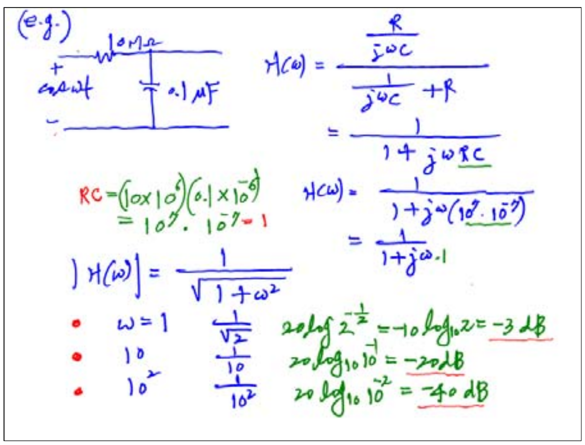




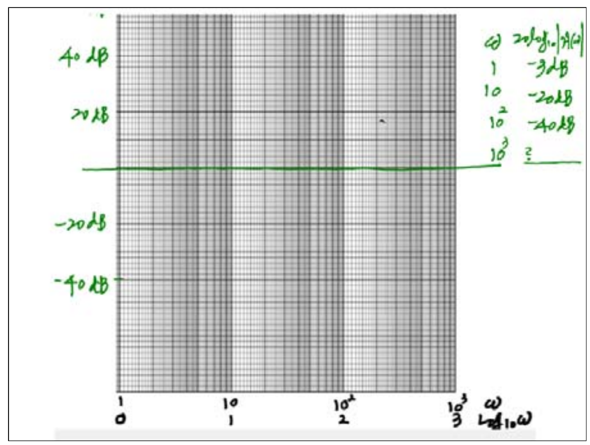
x(t) → H → y(t)
Y(ω) = H(ω) X(ω)
|Y(ω)|² = |H(ω)|² |X(ω)|²
10 log₁₀ |Y(ω)|² = 20 log₁₀ |H(ω)| + 20 log₁₀ |X(ω)|²
20 log₁₀ |H(ω)| [dB]
40
20
0
-20
10⁰ 10¹ 10² 10³
log₁₀ω



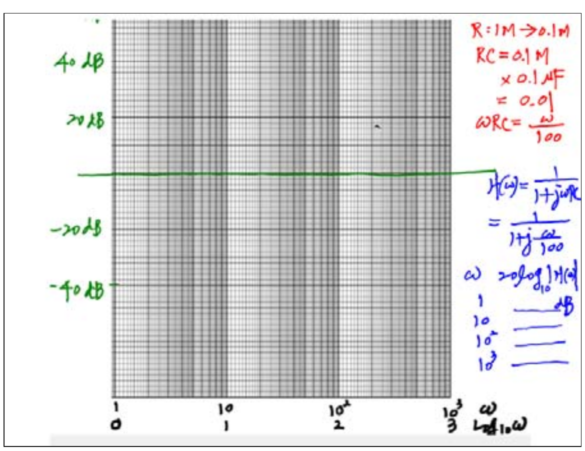
60 dB
40 dB
20 dB
-20 dB
-40 dB
1 10 10² 10³ ω
0 1 2 3 Log₁₀ω



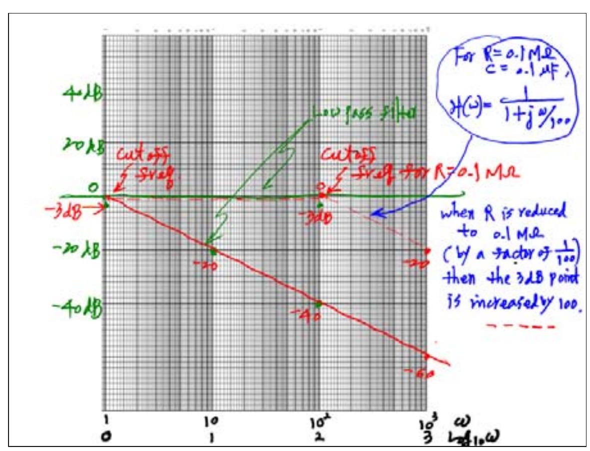
(e.g.)
10 MΩ
cos ωt
0.1 μF
H(ω) = (1/jωC) / (1/jωC + R)
= 1 / (1 + jωRC)
RC = (10×10⁶)(0.1×10⁻⁶) = 10⁷ · 10⁻⁷ = 1
H(ω) = 1 / (1 + jω(10⁷ · 10⁻⁷))
= 1 / (1 + jω·1)
|H(ω)| = 1 / √(1 + ω²)
ω = 1   1/√2   20 log 2^(-1/2) = -10 log₁₀ 2 = -3 dB
10   1/10   20 log₁₀ 10⁻¹ = -20 dB
10²   1/10²   20 log₁₀ 10⁻² = -40 dB



40 dB
20 dB
-20 dB
-40 dB
ω   20 log₁₀ |H(ω)|
1   -3 dB
10   -20 dB
10²   -40 dB
10³   ?
1 10 10² 10³ ω
0 1 2 3 Log₁₀ω



R: 1M → 0.1M
RC = 0.1 M × 0.1 μF = 0.01
ωRC = ω/100
H(ω) = 1/(1+jωRC) = 1/(1 + j ω/100)
ω   20 log₁₀ |H(ω)|
1   ___ dB
10
10²
10³
40 dB
20 dB
-20 dB
-40 dB
1 10 10² 10³ ω
0 1 2 3 Log₁₀ω

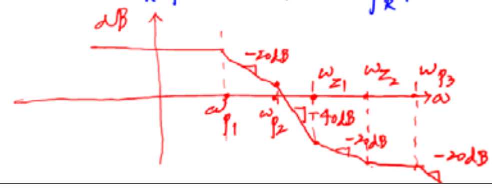


For R = 0.1 MΩ, C = 0.1 μF,
H(ω) = 1/(1 + j ω/100)
Low pass filter
cut off freq
cutoff freq for R = 0.1 MΩ
-3 dB
-3dB
-20
-40
-60
when R is reduced to 0.1 MΩ (by a factor of 1/100) then the 3dB point is increased by 100.
40 dB
20 dB
0
-20 dB
-40 dB
1 10 10² 10³ ω
0 1 2 3 Log₁₀ω

In general

$$H(\omega) = K \frac{\left(1+\frac{\omega}{\omega_{z_1}}\right)\left(1+\frac{\omega}{\omega_{z_2}}\right)\cdots\left(1+\frac{\omega}{\omega_{z_M}}\right)}{\left(1+\frac{\omega}{\omega_{p_1}}\right)\left(1+\frac{\omega}{\omega_{p_2}}\right)\cdots\left(1+\frac{\omega}{\omega_{p_N}}\right)}$$

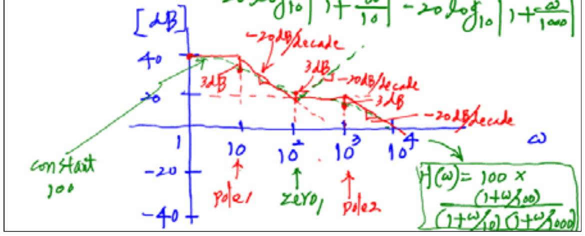
$$20\log_{10}|H(\omega)| = 20\log_{10}|K| + \sum_{j=1}^{M} 20\log_{10}\left|1+\frac{\omega}{\omega_{z_j}}\right| - \sum_{k=1}^{N} 20\log_{10}\left|1+\frac{\omega}{\omega_{p_k}}\right|$$

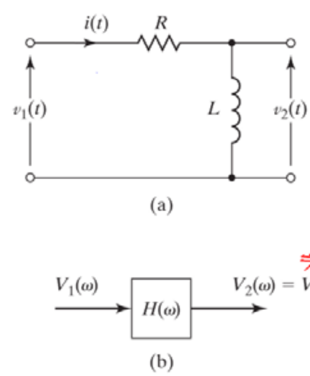


Example

$$H(\omega) = 100 \frac{\left(1+\frac{\omega}{100}\right)}{\left(1+\frac{\omega}{10}\right)\left(1+\frac{\omega}{1000}\right)}$$

$$20\log_{10}|H(\omega)| = \underbrace{20\log_{10}100}_{=40} + 20\log_{10}\left|1+\frac{\omega}{100}\right| - 20\log_{10}\left|1+\frac{\omega}{10}\right| - 20\log_{10}\left|1+\frac{\omega}{1000}\right|$$





$v_2(t) = L\frac{di(t)}{dt}$

$v_R(t) = R\,i(t)$

$\boxed{v_1(t) = L\frac{di(t)}{dt} + R\,i(t)}$

solve for $i(t)$

$V_1(\omega) = (j\omega L + R)I(\omega)$

and

$V_2(\omega) = \mathcal{F}\left[L\frac{di(t)}{dt}\right] = j\omega L\, I(\omega)$

$\Rightarrow V_2(\omega) = \underbrace{\frac{j\omega L}{R+j\omega L}}_{H(\omega)} V_1(\omega)$

$$|H(\omega)| = \frac{\omega L}{\sqrt{R^2+(\omega L)^2}}$$

$$\angle H(\omega) = 90^\circ - \tan^{-1}\frac{\omega L}{R}$$

$$|V_2(\omega)| = |V_1(\omega)|\,|H(\omega)|$$

$$\angle V_2(\omega) = \angle V_1(\omega) + \angle H(\omega)$$

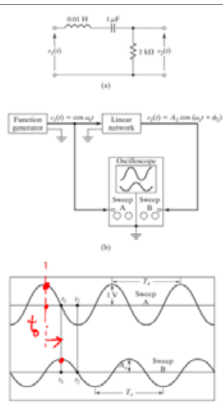
thus, $V_2(\omega) = |V_1(\omega)|\,|H(\omega)|\, e^{j\overbrace{(\phi_1(\omega) + \angle H(\omega))}^{\phi_2(\omega)}}$

For $v_1(t) = \cos\omega_x t$, $|V_1(\omega_x)| = 1$, $\phi_1 = 0^\circ$

and $v_2(t) = 1\cdot|H(\omega_x)|\cos(\omega_x t + \phi_2(\omega = \omega_x))$

$= A_2 \cos(\omega_x t + \phi_2)$

$\omega_x = 2\pi f_x = 2\pi/T_x$



$j\omega L - j\omega 10^{-2}$

$\frac{1}{j\omega C} = \frac{1}{j\omega 10^{-6}}$

$j\omega L - j\frac{1}{\omega C} = j\left(\omega L - \frac{1}{\omega C}\right) = 0$

$\omega L = \frac{1}{\omega C}$ $\omega = \frac{1}{\sqrt{LC}}$

$(\omega_x t + \phi_2) = \omega_x\left(t + \frac{\phi_2}{\omega_x}\right)$

$(t_1 - t_0) = \text{delay} = -\phi_2/\omega_x$

$\Rightarrow \phi_2 = -2\pi\frac{(t_1-t_0)}{T_x}$

$\omega_x = \frac{2\pi}{T_x}$