ECE 103 Fall 2018 Lecture 17 Nov 7, 2018

Midterm Average = 68.5

Qz avg 20%
midterm 30%
Final 50%

$$F(s) = \frac{1}{s}\,\frac{1}{s^2+s+1}$$

$$(s^2+s+1) = \left(s+\tfrac{1}{2}\right)^2 + \tfrac{3}{4}$$
$$= \left(s+\tfrac{1}{2}\right)^2 - j^2\left(\tfrac{\sqrt{3}}{2}\right)^2$$
$$= (\underbrace{s+\tfrac{1}{2}+j\tfrac{\sqrt{3}}{2}}_{\alpha})(s+\underbrace{\tfrac{1}{2}-j\tfrac{\sqrt{3}}{2}}_{\alpha^*})$$

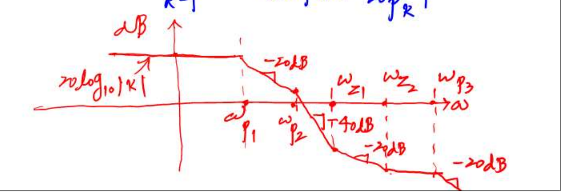
$$F(s) = \frac{1}{s}\,\frac{1}{(s+\alpha)(s+\alpha^*)}$$
$$= \frac{1}{s} + \frac{A}{s+\alpha} + \frac{A^*}{s+\alpha^*}$$

$A = ?\quad F(s)\,(s+\alpha)\Big|_{s=-\alpha}$

In general

$$H(\omega) = K\,\frac{\left(1+\frac{\omega}{\omega_{z_1}}\right)\left(1+\frac{\omega}{\omega_{z_2}}\right)\cdots\left(1+\frac{\omega}{\omega_{z_M}}\right)}{\left(1+\frac{\omega}{\omega_{p_1}}\right)\left(1+\frac{\omega}{\omega_{p_2}}\right)\cdots\left(1+\frac{\omega}{\omega_{p_N}}\right)}$$

$$20\log_{10}|H(\omega)| = 20\log_{10}|K| + \sum_{j=1}^{M} 20\log_{10}\left|1+\frac{\omega}{\omega_{z_j}}\right| - \sum_{k=1}^{N} 20\log_{10}\left|1+\frac{\omega}{\omega_{p_k}}\right|$$



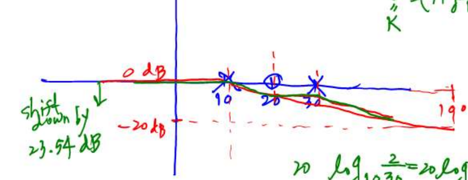
$$H(s) = \frac{(s+20)}{(s+10)(s+30)}$$

zero 1

pole 1 pole 2

$s \leftarrow j\omega$

$$s \leftarrow j\omega,\quad \frac{20}{10\cdot 30}\,\frac{\left(1+j\frac{\omega}{20}\right)}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{30}\right)} = \underbrace{\left[\frac{2}{30}\right]}_{K}\,\frac{\left(1+j\frac{\omega}{20}\right)}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{30}\right)}$$



$$20\log_{10}\frac{2}{30} = 20\log_{10}2 - 20\log_{10}30 \quad (=3\times10)$$
$$= 20(0.301) - 20\log_{10}3 - 20\log_{10}10$$
$$= 6 - 20\times 0.4771 - 20$$
$$= 6 - 9.54 - 20 = -23.54\ \text{dB}$$

---

at $\omega = 100$

$$\frac{2}{30}\,\frac{1+j5}{(1+j10)(1+j3.3)} = H(\omega=100)$$

$$|H| = \left|\frac{2}{30}\right|\,\frac{|1+j5|}{|(1+j10)|\,|1+j3.3|}$$

$$20\log_{10}|H| = \underbrace{20\log_{10}\frac{20}{30}}_{=-23.54\text{dB}} + 20\log_{10}\sqrt{26}$$
$$-\underbrace{20\log_{10}\underbrace{\sqrt{101}}_{\approx 10}}_{=20\text{dB}} - 20\log_{10}\sqrt{11.89}$$

$11.89 \approx 12 = 3\times 4$

3.3×3.3: 99, 99, 10.89

-36dB

$20(\log_{10}3 + \log_{10}4)$; 0.477, 0.602 ; 1.79

-23.54
-20
-36
-79.5
~25
-54 dB

$$\frac{(s+20)}{(s+10)(s+30)} \xrightarrow{s=j\omega} \frac{(j\omega+20)}{(j\omega+10)(j\omega+30)}$$

$$= \frac{20\left(1+j\frac{\omega}{20}\right)}{10\left(1+j\frac{\omega}{10}\right)30\left(1+j\frac{\omega}{30}\right)}$$

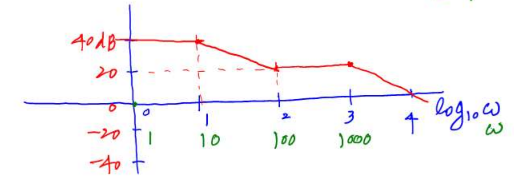
$$= \underbrace{\left[\frac{20}{10\times 30}\right]}_{K}\,\frac{\left(1+j\frac{\omega}{20}\right)}{\left(1+j\frac{\omega}{10}\right)\left(1+\frac{j\omega}{30}\right)}$$

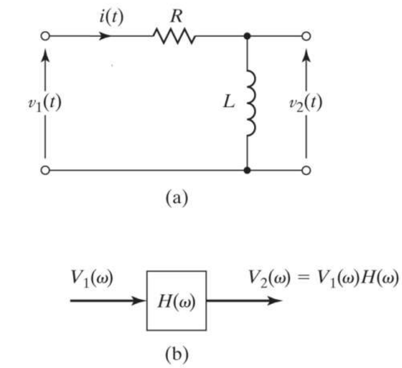
$$K\,\frac{\left(1+j\frac{\omega}{\omega_{z_1}}\right)}{\left(1+j\frac{\omega}{\omega_{p_1}}\right)\left(1+j\frac{\omega}{\omega_{p_2}}\right)}$$

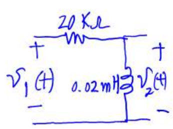
Example

$$\mathcal{H}(\omega) = 100 \frac{\left(1+\frac{j\omega}{100}\right)}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{1000}\right)}$$

$$20\log_{10}|\mathcal{H}(\omega)| = \underbrace{20\log_{10}100}_{=40} + 20\log_{10}\left|1+\frac{j\omega}{100}\right| - 20\log_{10}\left|1+\frac{j\omega}{10}\right| - 20\log_{10}\left|1+\frac{j\omega}{1000}\right|$$



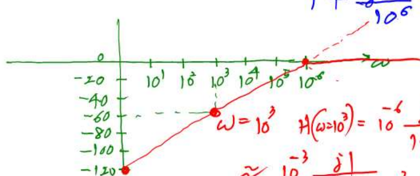




$$V_2(s) = \frac{0.02\times10^{-3}s}{20\times10^{3}+0.02\times10^{-3}s}V_1(s)$$

$$= 10^{-6}\frac{s}{1+10^{-6}s}V_1(s)$$

$$\mathcal{H}(\omega) = 10^{-6}\frac{j\omega}{1+\frac{j\omega}{10^6}}$$

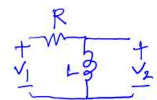


$\omega = 10^3$ $\quad \mathcal{H}(\omega=10^3) = 10^{-6}\frac{j10^3}{1+j\frac{10^3}{10^6}}$

$\approx 10^{-3}\frac{j1}{1}$? dB

$$|\mathcal{H}(\omega)| = \frac{\omega L}{\sqrt{R^2+(\omega L)^2}}$$

$$\angle\mathcal{H}(\omega) = 90^\circ - \tan^{-1}\frac{\omega L}{R}$$



$$\mathcal{H}(\omega) = \frac{j\omega L}{R+j\omega L}$$

$$V_2(\omega) = \underbrace{|V_2(\omega)|}\, e^{j\underbrace{\angle V_2(\omega)}=\phi_2}$$

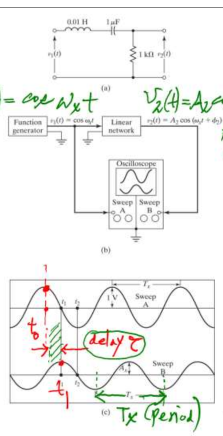
$$= V_1(\omega)\,\mathcal{H}(\omega) = |V_1(\omega)|e^{j\angle V_1(\omega)}|\mathcal{H}(\omega)|e^{j\angle\mathcal{H}(\omega)}$$

$$= \underbrace{|V_1(\omega)|\,|\mathcal{H}(\omega)|}\, e^{j(\underbrace{\angle V_1(\omega)}_{\phi_1} + \underbrace{\angle\mathcal{H}(\omega)}_{\phi_H})}$$

For $v_1(t) = \cos\omega_x t$, $|V_1(\omega)| = 1$, $\phi_1 = 0^\circ$

and $v_2(t) = \underbrace{1\cdot|\mathcal{H}(\omega_x)|}_{A_2}\cos(\omega_x t + \underbrace{(\phi_1+\phi_H(\omega_x))}_{=\phi_2})$

$= A_2\cos(\omega_x t + \phi_2)$



$v_1(t) = \cos\omega_x t \qquad v_2(t) = A_2\cos(\omega_x t + \phi_2)$

Here

$(\omega_x t + \phi_2)$

$= \omega_x\left(t + \frac{\phi_2}{\omega_x}\right)$

$= \omega_x\left(t - \underbrace{\left(-\frac{\phi_2}{\omega_x}\right)}_{=\tau}\right)$

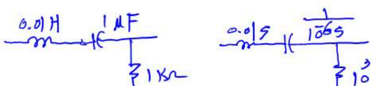
$(t_1 - t_0) = \tau$

$= -\frac{\phi_2}{\omega_x}$

$\phi_2 = -\omega_x(t_1 - t_0)$

$\overset{\uparrow}{=} -2\pi\frac{(t_1-t_0)}{T_x}$

$\omega_x T_x = 2\pi$



$$\mathcal{H}(s) = \frac{10^3}{0.01s + \frac{1}{10^{-6}s} + 10^3}$$

$$= \frac{10^{-3}s}{10^{-8}s^2+10^{-3}s+1}$$

$$= 10^5\frac{s}{s^2+10^5 s+10^8}$$

$$= 10^5\frac{s}{(s+\alpha_1)(s+\alpha_2)}$$

poles at $\alpha_1\,\alpha_2 = \frac{1}{2}\left(10^5 \pm \sqrt{(10^5)^2 - 4(1)(10^8)}\right)$

$= \frac{1}{2}\left(10^5 \pm \sqrt{10^{10} - 4\times10^8}\right)$

$= \frac{1}{2}\left(10^5 \pm 10^4\sqrt{96}\ (= 4\sqrt{6})\right)$

$as^2 + bs + c = 0$

$s = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$s^2 + 10^5 s + 10^8$

$s = \dfrac{-10^5 \pm \sqrt{(10^5)^2 - 4(1)(10^8)}}{2 \cdot 1}$ α_1, α_2

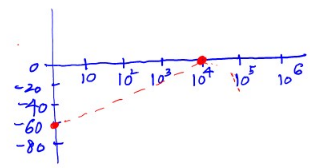
$(s - \alpha_1)(s - \alpha_2)$

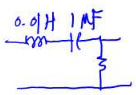
$10^5 \dfrac{s}{(s+\alpha_1)(s+\alpha_2)} \underset{s=j\omega}{\Rightarrow} 10^5 \dfrac{j\omega}{\alpha_1(1+\frac{j\omega}{\alpha_1})\,\alpha_2(1+\frac{j\omega}{\alpha_2})}$

$= \dfrac{10^5}{\underbrace{\alpha_1 \cdot \alpha_2}_{=10^8}} \dfrac{j\omega}{(1+j\frac{\omega}{\alpha_1})(1+j\frac{\omega}{\alpha_2})}$

$= 10^{-3} \dfrac{j\omega}{(1+j\frac{\omega}{\alpha_1})(1+j\frac{\omega}{\alpha_2})}$

$H(\omega = 10^4) = 1$





$j\omega L = 0.01\omega$

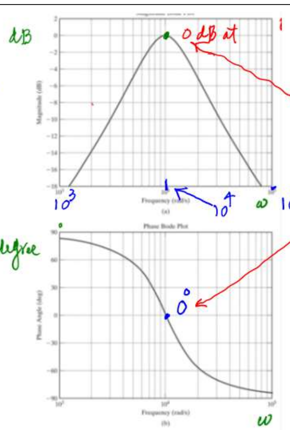
$\dfrac{1}{j\omega C} = \dfrac{1}{j\omega 10^{-6}}$

$j\omega L + \dfrac{1}{j\omega C} = 0$

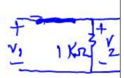
at $\omega = \dfrac{1}{\sqrt{LC}}$

$= \dfrac{1}{\sqrt{10^{-2} \cdot 10^{-6}}}$

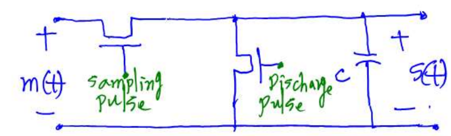
$= 10^4$

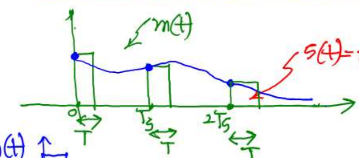


At $\omega = 10^4$

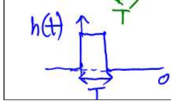


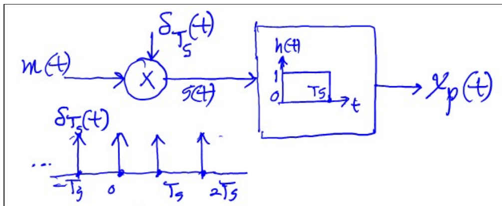
" Sample and Hold " Circuit





$s(t) = m(t) * h(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$





$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

$s(t) = m(t)\,\delta_{T_s}(t)$

$s(t) = h(t) * m(t)\,\delta_{T_s}(t)$

$= h(t) * m(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

Here $s(t) = m(t) * h(t)\,\delta_{T_s}(t)$

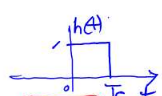
$\downarrow \mathcal{F}$ where $\delta_{T_s} = \sum \delta(t - nT_s)$

$S(\omega) = M(\omega)\left[H(\omega) * \Delta_{\omega_s}(\omega)\right]$

where $\Delta_{\omega_s} = \omega_s \sum \delta(\omega - n\omega_s)$

Table 5.2

$\omega_s T_s = 2\pi$ and $\omega_s = \frac{2\pi}{T_s}$



$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$

$= \int_0^{T} 1\, e^{-j\omega t}\, dt = \dfrac{e^{-j\omega t}}{-j\omega}\Bigg|_0^{T_s}$

$= \boxed{\dfrac{e^{-j\omega T_s} - 1}{-j\omega}} = \dfrac{1}{j\omega} e^{-j\omega\frac{T_s}{2}} \left(e^{+j\omega\frac{T_s}{2}} - e^{-j\omega\frac{T_s}{2}}\right) = T_s e^{-j\omega\frac{T_s}{2}} \dfrac{(\quad)}{2j\omega\frac{T_s}{2}}$

$= \left(e^{-j\omega\frac{T_s}{2}}\right) T_s \,\mathrm{sinc}\left(\omega \frac{T_s}{2}\right)$

$$\frac{e^{-j\omega T_s}-1}{-j\omega} = \frac{1-e^{-j\omega T_s}}{j\omega}$$

$$= \frac{e^{-j\omega\frac{T_s}{2}}\left(e^{j\omega\frac{T_s}{2}} - e^{-j\omega\frac{T_s}{2}}\right)}{2j\omega\frac{1}{2}}$$

$$= e^{-j\omega\frac{T_s}{2}}\, T_s \frac{\sin \omega\frac{T_s}{2}}{\omega\frac{1}{2}T_s}$$

$$= \underbrace{\left(e^{-j\omega\frac{T_s}{2}}\right)}_{\text{phase angle or delay}}\left(T_s \operatorname{sinc} \omega\frac{T_s}{2}\right)$$

$$\frac{e^{+j\theta} - e^{-j\theta}}{2j} = \sin\theta$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Recall

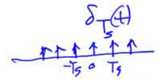
$$\delta(t) \xrightarrow{\mathcal{F}} \int_{-\infty}^{\infty}\delta(t)e^{-j\omega t}dt = 1$$

$$\boxed{\delta(t-t_0)} \longrightarrow \int_{-\infty}^{\infty}\delta(t-t_0)e^{-j\omega(t-t_0+t_0)}d(t-t_0)$$

$$= \int_{-\infty}^{\infty}\delta(\tau)e^{-j\omega\tau}e^{-j\omega t_0}d\tau$$

$$= \boxed{e^{-j\omega t_0}} = \mathcal{F}\left[\delta(t-t_0)\right]$$

$\delta_{T_s}(t) \longrightarrow$



$$\delta_{T_s}(t) = \sum c_n e^{+j\omega n T_s t}$$

$$\overline{\uparrow}\ \frac{1}{T_s}\sum e^{+j\omega n T_s t}$$

$$c_n = \frac{1}{T_s}$$

$$e^{j\omega_s t} \longrightarrow \boxed{2\pi\delta(\omega-\omega_s)}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}2\pi\delta(\omega-\omega_s)e^{j\omega t}d\omega \longleftarrow 2\pi\,\delta(\omega-\omega_s)$$

$$= \int_{-\infty}^{\infty}e^{j\omega_s t}\delta(\omega-\omega_s)d\omega$$

$$= e^{j\omega_s t}$$

$$= \frac{1}{j\omega}e^{-j\omega\frac{T_s}{2}}\left(e^{+j\omega\frac{T_s}{2}} - e^{-j\omega\frac{T_s}{2}}\right)$$

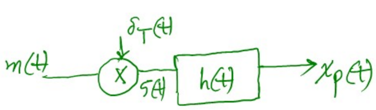
$$= \frac{1}{2j\frac{\omega}{2}T_s}T_s\, e^{-j\omega\frac{T_s}{2}}\left(e^{+j\omega\frac{T_s}{2}} - e^{-j\omega\frac{T_s}{2}}\right)$$

$$= T_s e^{-j\omega\frac{T_s}{2}}\left(\frac{\sin\omega\frac{T_s}{2}}{\omega\frac{T_s}{2}}\right) = \operatorname{sinc}\omega\frac{T_s}{2}$$

$$= T_s e^{-j\omega\frac{T_s}{2}} \operatorname{sinc}\omega\frac{T_s}{2} = H(\omega)$$

where $T_s = 2\pi/\omega_s$

$$\Rightarrow H(\omega) = T_s e^{-j\omega\frac{\pi}{\omega_s}} \operatorname{sinc}\omega\frac{\pi}{\omega_s} \qquad (*)$$



$$X_p(\omega) = H(\omega)\sum_{n=-\infty}^{\infty}m(nT_s)e^{-j\omega nT_s}$$

$$X_p(\omega) = H(\omega)\sum m(nT_s)e^{-j\omega nT_s}$$

$$= \left(T_s e^{-j\pi\frac{\omega}{\omega_s}}\operatorname{sinc}\left(\pi\frac{\omega}{\omega_s}\right)\right)\sum m(nT_s)e^{-j\omega nT_s}$$

$$= T_s\left[e^{-j\pi\frac{\omega}{\omega_s}}\operatorname{sinc}\left(\pi\frac{\omega}{\omega_s}\right)\right]\sum m(nT_s)e^{-j\omega nT_s}$$

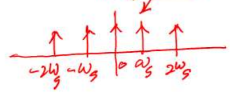
$$\sum e^{-j\omega nT_s}$$

$$= T_s\underbrace{\left(\frac{1}{T_s}\sum e^{+j\omega nT_s}\right)}_{\text{Fourier Series}}$$

$$= T_s\,\delta_{T_s}(\omega-\omega_s)$$

$$= \left(2\pi\frac{T_s}{2\pi}\right)\delta_{T_s}(\omega-\omega_s)$$

$$= 2\pi\,\omega_s\,\delta_{T_s}(\omega-\omega_s)$$

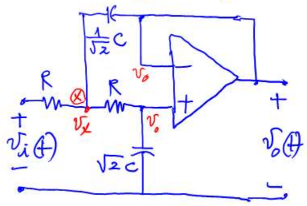


Recall $\mathcal{F}^{-1}\left[2\pi\delta(\omega-\omega_s)\right]$

$$= \cancel{\frac{1}{2\pi}}\int\cancel{2\pi}\,\delta(\omega-\omega_s)e^{+j\omega t}d\omega$$

$$= e^{j\omega_s t}$$

Prob 6.10 (Fig P6.10)



At node ⊗, KCL is

$$\frac{1}{\sqrt{2}} C s (V_x - V_o) + \frac{1}{R}(V_x - V_o) + \frac{1}{R}(V_x - V_i) = 0 \quad (1)$$

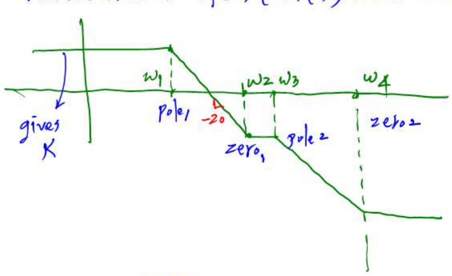
$$V_x = R\sqrt{2} C s\, V_o + V_o = (1+\sqrt{2} RCs) V_o \quad (2)$$

$$= \frac{1}{\sqrt{2}} Cs\left((1+\sqrt{2}RCs)V_o - V_o\right) + \frac{1}{R}\left((1+\sqrt{2}RCs)V_o - V_o\right) + \frac{1}{R}\left((1+\sqrt{2}RCs)V_o - V_i\right) = 0$$

$$\Rightarrow \frac{1}{\sqrt{2}} RCs\left(\sqrt{2}RCs V_o\right) + \sqrt{2}RCs V_o + (1+\sqrt{2}RCs)V_o = V_i$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{(RCs)^2 + 2\sqrt{2}RCs + 1} = \frac{1}{(RC)^2}\,\frac{1}{s^2 + \frac{2\sqrt{2}}{RC}s + \left(\frac{1}{RC}\right)^2} = A\,\frac{1}{(s+\alpha_1)(s+\alpha_2)}$$

Derivation of $H(\omega)$ (or $H(s)$) from Bode plot



$$H(\omega) = K\frac{\left(1+j\frac{\omega}{\omega_2}\right)\left(1+j\frac{\omega}{\omega_4}\right)}{\left(1+j\frac{\omega}{\omega_1}\right)\left(1+j\frac{\omega}{\omega_3}\right)} \longrightarrow H(s) = K\frac{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_4}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_3}\right)}$$