ECE 103 F18 Lecture 2 Oct 1, 2018

HW #1 (Quiz 1 on Oct 8 at 3:30 p.m.)

[1] Textbook 5th ed / 4th ed
Prob 2.1(a) (i), (iii) / 2.1(a) (i), (iii)

[2] prob 2.5(a) / 2.7(a)

[3] prob. 2.12(a) / prob 2.14 with $x(t) = 5\sin(15t - 60°) + 2\sin(7t)$

[4] prob 2.16 (b) / $x(t) = 4\cos 4\pi t + 3\sin 4\pi t = A\cos(\omega_0 t + \phi)$
Find A, ω_0, ϕ

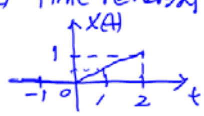
[5] prob 2.25(a) / prob 2.18(a)

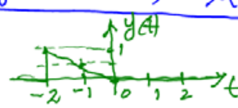
References/Books on Signals & Systems

- http://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/lecture-notes/ in particular lectures 1–9, 12–14, 16, 20–21, 24 by Prof. Alan Oppenheim
- Signal and Linear System Analysis, Gordon E. Carlson, John Wiley & Sons
- Linear Systems and Signals, B.P. Lathi & R. Green, Oxford University Press
- Continuous and Discrete Signals and Systems, S.S. Soliman and M.D. Srinath, Pearson

Transformation of continuous-time signals

(I) Time reversal $\boxed{y(t) = H(x(t)) = x(-t)}$





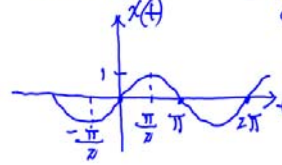
t	$x(t)$
$t<0$	0
0	0
1	$\frac{1}{2}$
2	1
$t>2$	0

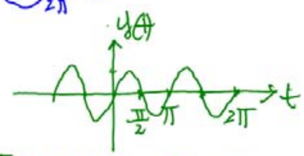
Mirror image w.r.t $t=0$ axis

t	$y(t) = x(-t)$
0	$x(-0) = x(0) = 0$
1	$x(-1) = 0$
2	$x(-2) = 0$
-1	$x(-(-1)) = x(1) = \frac{1}{2}$
-2	$x(-(-2)) = x(2) = 1$
$t<-2$	$x(>2) = 0$

(II) Time scaling $\boxed{y(t) = x(at)}$

$x(t) = \sin t$ $\quad y(t) = \sin 2t$, $a = 2$





For $a>1$, waveform is compressed w.r.t time

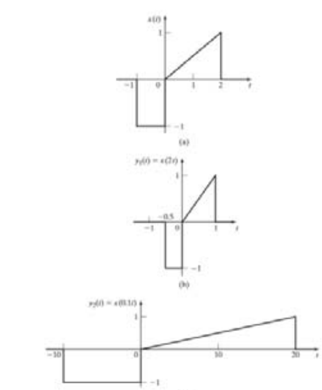
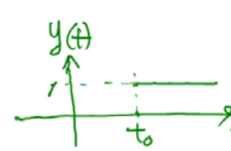


Figure 2.2 Time-scaled signals.

(III) Time shifting $\boxed{y(t) = x(t - t_0)}$

$x(t) = u(t) = 1(t)$



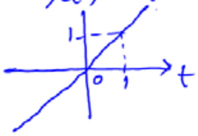
$t<0$ $\quad x(t) = 0$
$t>0$ $\quad x(t) = 1$

$t<t_0$ $\quad (t-t_0)<0$ $\quad x(t-t_0) = 0$
$t>t_0$ $\quad (t-t_0)>0$ $\quad x(t-t_0) = 1$

waveform is shifted on time axis by t_0
If $t_0>0$, to right
If $t_0<0$, to left

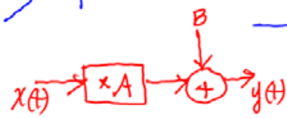
(III) Amplitude Transformation $\boxed{y(t)=Ax(t)+B}$

$x(t) = t$



$y(t) = 2t+1$

$A=2,\ B=1$



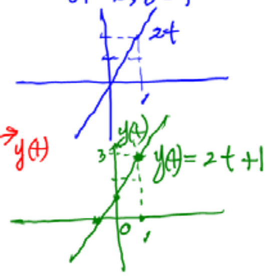
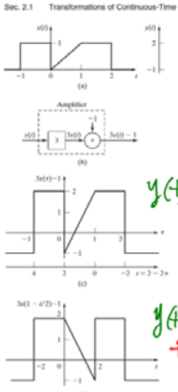


TABLE 2.1 Transformations of Signals

Name	$y(t)$
Time reversal	$x(-t)$
Time scaling	$x(at)$
Time shifting	$x(t-t_0)$
Amplitude reversal	$-x(t)$
Amplitude scaling	$Ax(t)$
Amplitude shifting	$x(t)+B$

Combined Time and Amplitude Transformation

Sec. 2.1 Transformations of Continuous-Time Signals



$y(t) = 3x(t)-1$
amplitude transformation

$y(t) = 3x(1-\frac{t}{2})-1$
time transformation added

Figure 2.9 Signals for Examples 2.2 and 2.3.

Signal characteristics $x(t) = x_e(t) + x_o(t)$

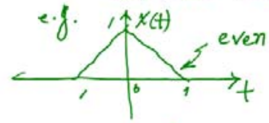
(I) Even signals $\boxed{x_e(-t) = x_e(t)}$

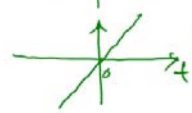
e.g. $x(t) = \cos\omega t$

$x(-t) = \cos\omega(-t) = \cos(-\omega t) = \cos\omega t$

thus $\cos\omega t$ is even

e.g.





not even

(II) odd signal $x_o(-t) = -x_o(t)$

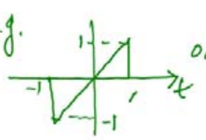
or $\boxed{x_o(t) = -x(-t)}$

e.g., $x(t) = \sin\omega t$

$x(-t) = \sin\omega(-t) = \sin(-\omega t) = -\sin\omega t = -x(t)$

$x(-t) = -x(t)$ thus $x(t)$ is odd.

e.g.



odd

How to obtain $x_e(t), x_o(t)$ of $x(t)$?

$x(t) = x_e(t) + x_o(t)$

$x(-t) = x_e(-t) + x_o(-t)$

$= x_e(t) - x_o(t)$

- $x(t) + x(-t) = 2x_e(t)$

$\Rightarrow \boxed{x_e(t) = \frac{1}{2}(x(t)+x(-t))}$

- $x(t) - x(-t) = 2x_o(t)$

$\Rightarrow \boxed{x_o(t) = \frac{1}{2}(x(t)-x(-t))}$

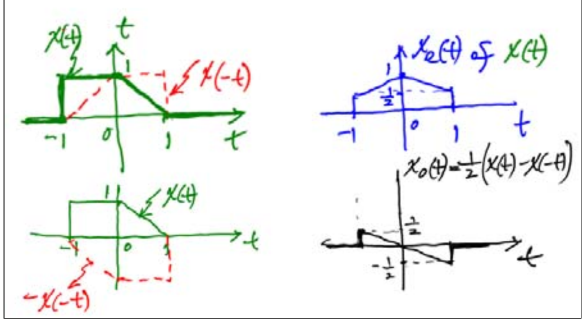
e.g. $x(t) = \cos\omega t + \sin\omega t$

- Find $x_e(t)$ of $x(t)$

$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$
$$= \frac{1}{2}\left((\cos\omega t + \sin\omega t) + (\cos\omega(-t) + \sin\omega(-t))\right)$$
$$= \frac{1}{2}\left(\cos\omega t + \cancel{\sin\omega t} + \cos\omega t - \cancel{\sin\omega t}\right)$$
$$= \underline{\cos\omega t}$$

- Find $x_o(t)$ of $x(t)$

$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$
$$= \frac{1}{2}\left((\cos\omega t + \sin\omega t) - (\cos\omega(-t) + \sin\omega(-t))\right)$$
$$= \frac{1}{2}\left(\cancel{\cos\omega t} + \sin\omega t - \cancel{\cos\omega t} - (-\sin\omega t)\right) = \underline{\sin\omega t}$$



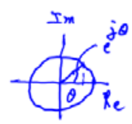
signals / unit complex numbers

$j = \sqrt{-1}$

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

Euler's formula

$$M = \sqrt{a^2 + b^2}$$
$$\theta = \tan^{-1}\frac{b}{a}$$



$$Z = a + jb = Me^{j\theta}$$

$z_1 = a + jb$

$z_2 = c + jd$

$$z_1 z_2 = (a+jb)(c+jd) = M_1 e^{j\theta_1} M_2 e^{j\theta_2} = M_1 M_2 e^{j(\theta_1+\theta_2)}$$

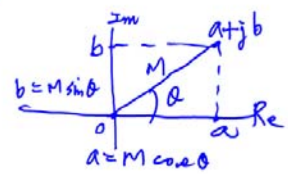
$$z_1/z_2 = M_1/M_2\, e^{j(\theta_1-\theta_2)}$$

$$\frac{z_1}{z_2} = \frac{a+jb}{c+jd} = \frac{M_1}{M_2} e^{j(\theta_1-\theta_2)}$$

Euler's formula

$$Me^{j\theta} = M(\cos\theta + j\sin\theta) = M\cos\theta + jM\sin\theta$$
$$= a + jb$$

$$M = \sqrt{a^2+b^2}$$
$$\theta = \tan^{-1}\frac{b}{a}$$



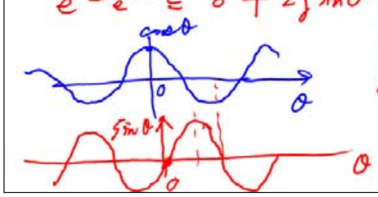
$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos(-\theta) + j\sin(-\theta)$$
$$= \cos\theta - j\sin\theta$$

$$e^{j\theta} + e^{-j\theta} = 2\cos\theta + 0 \qquad \boxed{\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)}$$

even function of θ

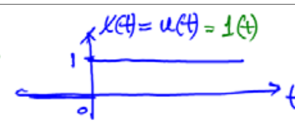
$$e^{j\theta} - e^{-j\theta} = 0 + 2j\sin\theta \qquad \boxed{\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)}$$

odd function of θ



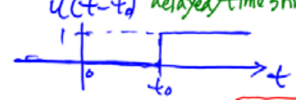
signals

u(t)

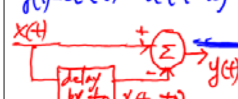


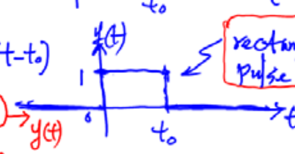
$\boxed{\text{unit step function}}$

$u(t - t_0)$ delayed/time shifted by t_0

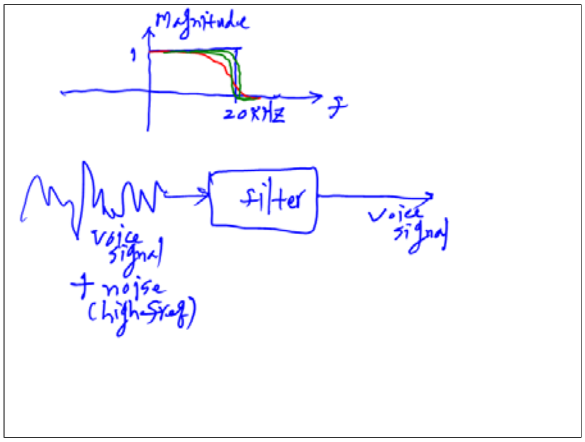


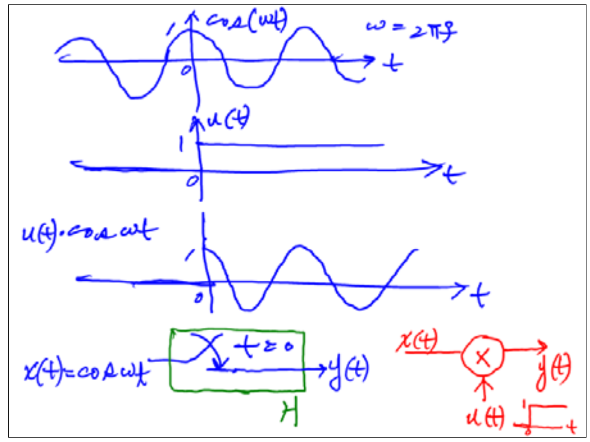
$$y(t) = u(t) - u(t - t_0)$$

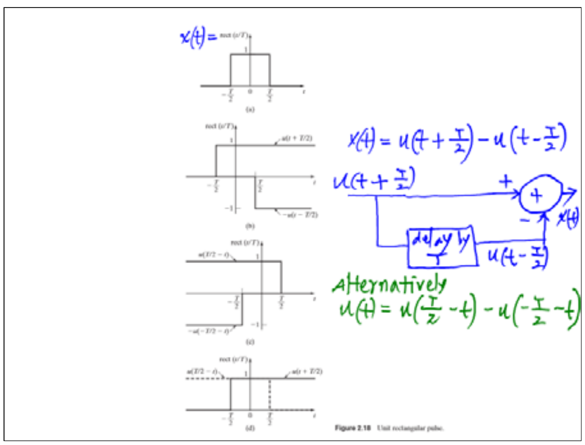


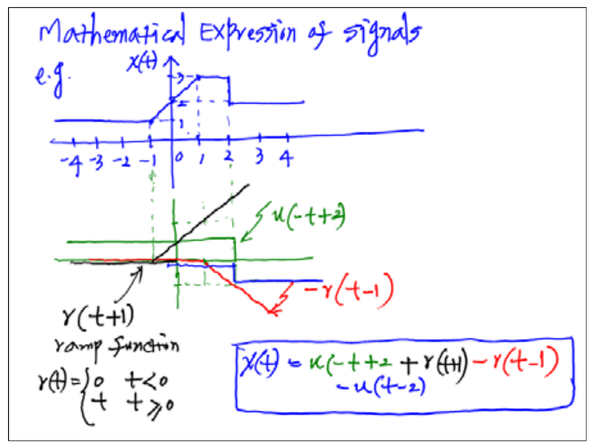


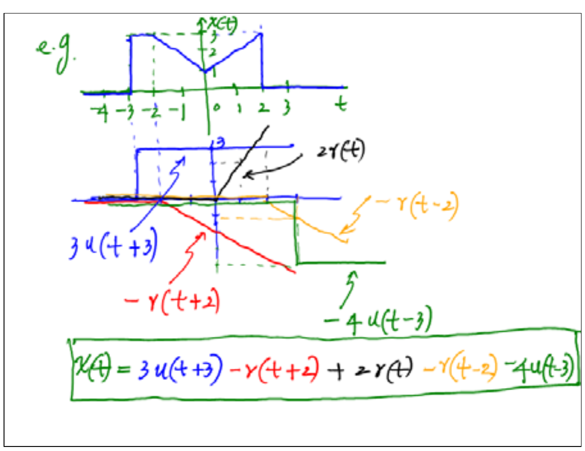
$\boxed{\text{rectangular pulse}}$


Magnitude
1
20KHZ
f
filter
voice signal
+ noise (high freq)
voice signal


cos(ωt)
ω = 2πf
t
u(t)
u(t)·cos ωt
x(t)=cos ωt
t=0
y(t)
H
x(t)
y(t)
u(t)


x(t) = rect (t/T)
x(t) = u(t + T/2) − u(t − T/2)
u(t + T/2)
delay by T
u(t − T/2)
x(t)
Alternatively
u(t) = u(T/2 − t) − u(−T/2 − t)
Figure 2.16 Unit rectangular pulse.


Mathematical Expression of signals
e.g.
x(t)
-4 -3 -2 -1 0 1 2 3 4
u(−t+2)
−r(t−1)
r(t+1)
ramp function
r(t) = 0 t<0; t t≥0
x(t) = u(−t+2) + r(t+1) − r(t−1) − u(t−2)


e.g.
x(t)
-4 -3 -2 -1 0 1 2 3
t
2r(t)
−r(t−2)
3u(t+3)
−r(t+2)
−4u(t−3)
x(t) = 3u(t+3) − r(t+2) + 2r(t) − r(t−2) − 4u(t−3)