

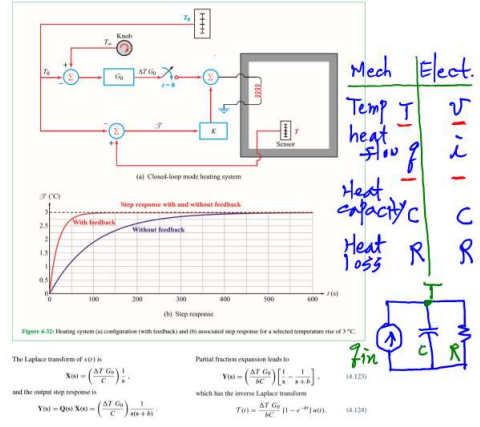
ECE 103 Lecture 24, Nov. 28, 2018

Quiz 8 today (this is the last quiz)
Midterm average = 69.3 $\sigma = 16.3$

Final Exam on Dec. 12 (w) 4-7 PM

- It will cover the entire course materials
- You are allowed to bring 2 pages of formulas only. No solved problems

Course score = $2 \times (\text{Average of best 8 quizzes}) + 0.3 \text{ Midterm} + 0.5 \text{ Final}$



Physical systems
Electrical, Mechanical, Fluidic

QUANTITY	PROTOTYPE	ELECTRICAL	TRANSLATIONAL	ROTATIONAL	FLUID-FLOW
Flow	Flow, f	Current, i	Force, F	Torque, τ	Volume flow, Q
Potential	Potential, ϕ	Voltage, v	Velocity, v	Angular vel., ω	Pressure, p
Flow equation	$f = Ys$	$i = Yv$	$F = Yv$	$\tau = Y\omega$	$Q = Yp$
Integratable power (watt)	$\phi = \int f dt$	$v = \int i dt$	$v = \int F dt$	$\omega = \int \tau dt$	$p = \int Q dt$
Other variables	Charge, $q = \int i dt$	Displacement, $x = \int v dt$	Angle, $\theta = \int \omega dt$		
Integrating constant, $\int_0^t f dt$	$\int_0^t i dt = q$	$\int_0^t v dt = x$	$\int_0^t F dt = \Delta p$	$\int_0^t \tau dt = \Delta \theta$	$\int_0^t Q dt = \Delta V$
Flow relation	$f = Ys \phi$	$i = Ys v$	$F = Ys v$	$\tau = Ys \omega$	$Q = Ys p$
\rightarrow Admittance	$Y = f/v$	$Y = i/v$	$Y = F/v$	$Y = \tau/\omega$	$Y = Q/p$
\rightarrow Impedance	$Z = v/f$	$Z = v/i$	$Z = v/F$	$Z = \omega/\tau$	$Z = p/Q$
Stored energy (joule)	ϕ	$\frac{1}{2} C i^2$	$\frac{1}{2} M v^2$	$\frac{1}{2} I \omega^2$	$\frac{1}{2} \rho Q^2$
Initial condition	$\phi(0)$	$i(0)$ or $q(0)$	$v(0)$ or $x(0)$	$\omega(0)$ or $\theta(0)$	$p(0)$
Proportional element, $\int_0^t f dt$	$\frac{1}{s}$	$\frac{1}{s}$	$\frac{1}{s}$	$\frac{1}{s}$	$\frac{1}{s}$
Flow equation	$f = \phi/s$	$i = v/s$	$F = v/s$	$\tau = \omega/s$	$Q = p/s$
\rightarrow Admittance	$Y = f/v$	$Y = i/v$	$Y = F/v$	$Y = \tau/\omega$	$Y = Q/p$
\rightarrow Impedance	$Z = v/f$	$Z = v/i$	$Z = v/F$	$Z = \omega/\tau$	$Z = p/Q$
Power (watt)	$\phi = \int f dt$	$v = \int i dt$	$F = \int v dt$	$\tau = \int \omega dt$	$p = \int Q dt$
Differentiating element, $\int_0^t f dt$	s	s	s	s	s
Flow equation	$f = s\phi$	$i = sv$	$F = sv$	$\tau = s\omega$	$Q = sp$
\rightarrow Admittance	$Y = f/v$	$Y = i/v$	$Y = F/v$	$Y = \tau/\omega$	$Y = Q/p$
\rightarrow Impedance	$Z = v/f$	$Z = v/i$	$Z = v/F$	$Z = \omega/\tau$	$Z = p/Q$
Stored energy (joule)	ϕ	$\frac{1}{2} C v^2$	$\frac{1}{2} M v^2$	$\frac{1}{2} I \omega^2$	$\frac{1}{2} \rho Q^2$
Initial condition	$\phi(0)$	$v(0)$ or $\phi(0)$	$v(0)$	$\omega(0)$	$p(0)$

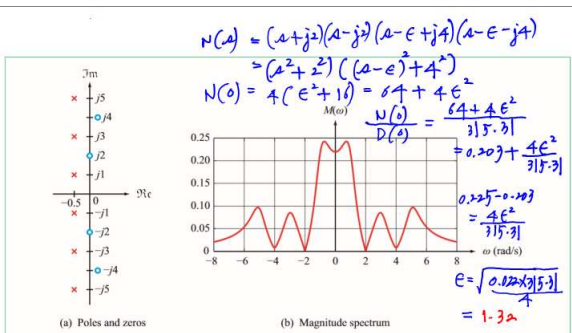
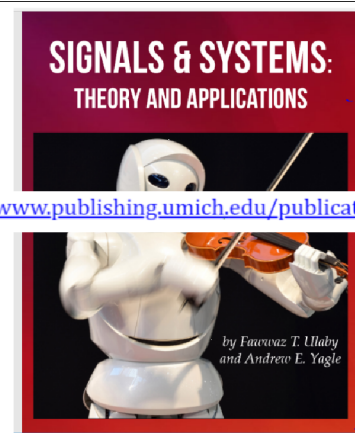


Figure 6-25: Magnitude spectrum of a frequency response composed of 2 pairs of conjugate zeros and 3 pairs of conjugate poles (Example 6-6).

$D(s) = (s+1.5+j1)(s+1.5-j1)(s+0.5+j3)(s+0.5-j3)(s+0.5+j5)(s+0.5-j5)$
 $= (s+0.9+1)(s+0.9+5)(s+0.9+5)$
 $P(0) = 1.25 \times 1.25 \times 25 = 315.31$

$H(s) = \frac{(s+z_1)(s+z_2) \dots}{(s+p_1)(s+p_2) \dots}$
 $(0.25 \rightarrow 1.0)$

⇒ some errors in this example

Perhaps

$$H(\omega) = \frac{(a+j2)(a-j2)(a+j4)(a-j4)}{(a+0.5+j1)(a+0.5-j1)(a+j2)(a-j2)}$$

$$= \frac{(a^2+4)(a^2+16)}{(a+0.5+j1)(a+0.5-j1)(a+j2)(a-j2)}$$

$$h(\omega) = \lim_{a \rightarrow \infty} a H(\omega) = \lim_{a \rightarrow \infty} \frac{a^4}{a^4} = 0$$

$$h(\omega) = \lim_{a \rightarrow 0} a H(\omega) = \lim_{a \rightarrow 0} \frac{a(4)(16)}{(0.5-j1)(0.5+j1)(2-j2)} = 0$$

$-0.5t$
 $e^{-0.5t}$ rejected →

Exercise 6-8: An LTI system has zeros at $\pm j3$. What sinusoidal signals will it eliminate?

Answer: $A \cos(3t + \theta)$ for any constant A and θ . (See (5)) which includes $A \sin 3t$

Exercise 6-9: An LTI system has poles at $-0.1 \pm j4$. What sinusoidal signals will it emphasize?

Answer: $A \cos(4t + \theta)$ for any constant A and θ . (See (5))

$$\frac{N(s)}{(s+0.1+j4)(s+0.1-j4)} = \frac{1}{(s+0.1)^2 + 4^2}$$

→ $e^{-0.1t} \cos(4t + \theta)$

Figure 6-27: Two attempts at designing a filter to reject $\omega_0 = \pm 120\pi$ (rad/s), i.e., $f_0 = 60$ Hz.

(b) Second attempt: poles and zeros

where in the last step, we applied the division relationship given by Eq. (3.55) to convert $H(s)$ from a proper rational function (numerator and denominator of the same degree) into a form $H_{\text{resch}} = 1 + G(s)$, where $G(s)$ is a strictly proper rational function (degree of numerator smaller than that of denominator). Application of partial fraction expansion followed by transformation to the time domain leads to the impulse response:

$$h_{\text{resch}}(t) = \delta(t) - Ae^{-\alpha t} \cos(\omega_0 t + \theta) \quad (6.77)$$

$$A = \left[4\alpha^2 + \omega_0^2 \right]^{1/2}, \quad \text{and } \theta = \tan^{-1} \left(\frac{\alpha}{2\omega_0} \right) \quad (6.78)$$

$$\frac{2d^2 + d^2}{(s+d)^2 + \omega_0^2} = \frac{2d(s+d) - d^2}{(s+d)^2 + \omega_0^2}$$

$$= 2d \left[\frac{(s+d)}{(s+d)^2 + \omega_0^2} - \frac{d^2}{\omega_0^2} \frac{1}{(s+d)^2 + \omega_0^2} \right]$$

$$= e^{-dt} \left[\frac{2d}{A} \cos \omega_0 t - \frac{d^2}{A} \sin \omega_0 t \right]$$

$$= B \cos(\omega_0 t + \theta)$$

$$\frac{d^2}{2d} \tan \theta = \frac{d^2}{2d}$$

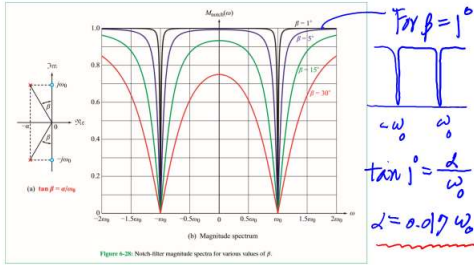


Figure 6-28. Match filter magnitude spectra for various values of β .

Example 6-7: Rejecting an Interfering Tone
 Design a notch filter to reject a 1 kHz interfering sinusoid. The filter's impulse response should decay to less than 0.01 within 0.1 s.
Solution: At 1 kHz, $\omega_0 = 2\pi \times 10^3 = 6283$ rad/s.
 To satisfy the impulse-response decay requirement, the value of α should be such that the coefficient of the cosine term in Eq. (6-77) is no larger than 0.01 at $t = 0.1$ s. That is,
 $(\alpha e^{-\alpha t})^{0.1} \leq 0.01$

Solution of this inequality (using MATLAB or MathScript) leads to $\alpha = 100 \text{ s}^{-1}$ and, in turn, to
 $\beta = \tan^{-1} \left(\frac{\alpha}{\omega_0} \right) = \tan^{-1} \left(\frac{100}{6283} \right) = 0.9^\circ$.
 With both ω_0 and β specified, we now can calculate and plot the impulse response given by Eq. (6-77) (including the impulse function), as shown in Fig. 6-29.
 Note that although the maximum value of $h_{\text{max}}(t)$, excluding the impulse, is 200, $|h_{\text{max}}(0)| = 0.00$, which is smaller than the specified value of 0.01. The magnitude spectrum of this notch filter is essentially the $\beta = 1^\circ$ plot shown in Fig. 6-28(b).

Concept Question 6-8: Why do we need poles in a notch filter? Why not just use zeros only? (See Q2)

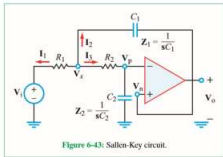


Figure 6-43. Sallen-Key circuit.

- (a) Obtain the transfer function $H(s) = V_o(s)/V_s(s)$.
- (b) Obtain the transfer function of a second-order Butterworth filter with cutoff frequency $\omega_c = 10^3$ rad/s and arbitrary dc gain C.
- (c) Match the transfer function in (a) and (b) to specify the values of R_1 , R_2 , C_1 , and C_2 .

Solution:
 (a) At node V_1 , KCL gives
 $\frac{V_s - V_1}{R_1} + \frac{V_2 - V_1}{R_2} + \frac{V_1 - V_0}{Z_1} = 0$, (6.102)
 and since $V_0 = V_a = V_p$, voltage division gives
 $V_1 = V_0 \left(\frac{R_2 + Z_2}{Z_2} \right)$. (6.103)

The combination of the two equations leads to
 $H(s) = \frac{1}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} \right) s + \frac{1}{R_1 R_2 C_1 C_2}}$. (6.104)

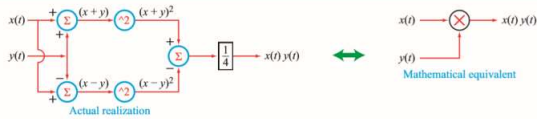


Figure 6-54. Block diagram realization of the product $x(t)y(t)$. The symbol \otimes denotes a squaring operation.

Handwritten notes:
 $I_{D5} = \frac{R_2}{2} (V_{q5} - V_{th})^2$
 $I_{D5} = \frac{R_2}{2} (V_{q5} - V_{th})^2$

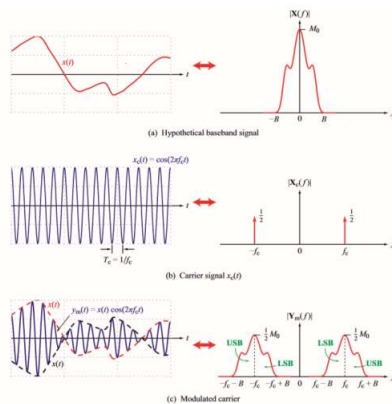


Figure 6-56. Bandpass signal $x(t)$, carrier signal $x_c(t)$, and modulated carrier $x_m(t)$.

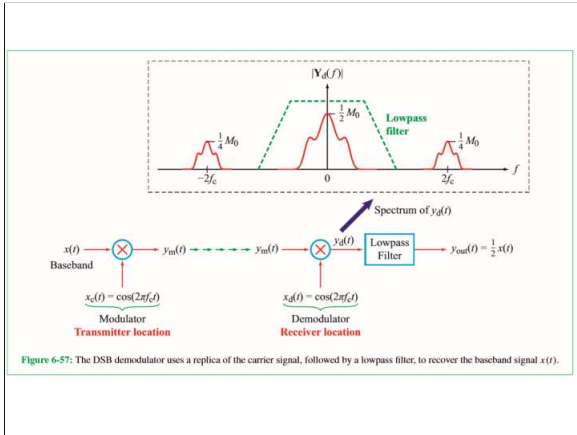


Table 5-4: Fourier series expressions for a select set of periodic waveforms.

Waveform	Fourier Series
1. Square Wave	$x(t) = \sum_{n=1}^{\infty} \frac{4A}{n\pi} \sin(n\omega_0 t) \cos\left(\frac{2n\pi\tau}{T_0}\right)$
2. Time-Shifted Square Wave	$x(t) = \sum_{n=1}^{\infty} \frac{8A}{n^2\pi^2} \sin\left(\frac{2n\pi\tau}{T_0}\right) \cos(n\omega_0 t)$
3. Pulse Train	$x(t) = \frac{At}{T_0} \sum_{n=1}^{\infty} \frac{2A}{n^2\pi^2} \sin\left(\frac{n\pi\tau}{T_0}\right) \cos(n\omega_0 t)$
4. Triangular Wave	$x(t) = \sum_{n=1}^{\infty} \frac{8A}{n^3\pi^3} \sin\left(\frac{2n\pi\tau}{T_0}\right) \cos(n\omega_0 t)$
5. Shifted Triangular Wave	$x(t) = \sum_{n=1}^{\infty} \frac{8A}{n^3\pi^3} \sin\left(\frac{n\pi\tau}{T_0}\right) \cos(n\omega_0 t)$
6. Sawtooth	$x(t) = \sum_{n=1}^{\infty} \frac{8A}{n^2\pi^2} \frac{2A}{n} \sin\left(\frac{2n\pi\tau}{T_0}\right) \cos(n\omega_0 t)$
7. Backward Sawtooth	$x(t) = \frac{A}{2} \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2} \sin\left(\frac{2n\pi\tau}{T_0}\right) \cos(n\omega_0 t)$
8. Full-Wave Rectified Sine	$x(t) = \frac{2A}{\pi} \sum_{n=1}^{\infty} \frac{4A}{n^2\pi^2} \cos(n\omega_0 t)$
9. Half-Wave Rectified Sine	$x(t) = \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{2A}{n^2\pi^2} \cos(n\omega_0 t) + \sum_{n=1}^{\infty} \frac{2A}{n^3\pi^3} \sin(n\omega_0 t) \cos\left(\frac{2n\pi\tau}{T_0}\right)$

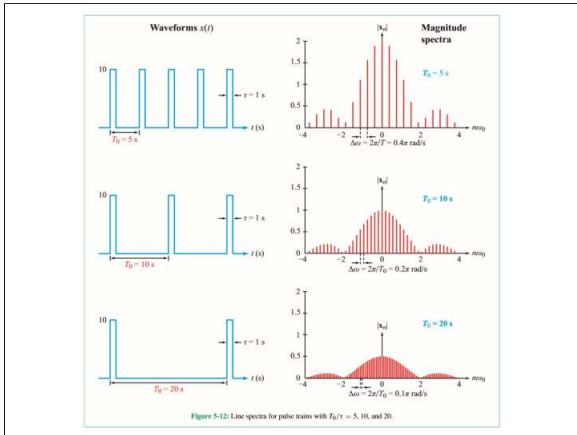


Table 5-4: Examples of Fourier transform pairs. Note that constant $a \geq 0$.

$x(t)$	$X(\omega) = \mathcal{F}\{x(t)\}$	$x(t)$	$X(\omega)$
BASIC FUNCTIONS			
1. $\frac{1}{T_0} \text{rect}\left(\frac{t}{T_0}\right)$	$\text{sinc}\left(\frac{\omega T_0}{2\pi}\right)$	1	$2\pi \delta(\omega)$
2. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	e^{-at}	$\frac{1}{1 + j\omega a}$
3. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\cos(\omega t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
4. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\sin(\omega t)$	$\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
5. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\cos(\omega t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
6. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\sin(\omega t)$	$\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
7. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\cos(\omega t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
8. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\sin(\omega t)$	$\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
9. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\cos(\omega t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
10. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\sin(\omega t)$	$\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
ADDITIONAL FUNCTIONS			
11. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\cos(\omega t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
12. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\sin(\omega t)$	$\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
13. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\cos(\omega t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
14. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\sin(\omega t)$	$\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
15. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\cos(\omega t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
16. $\frac{1}{T_0} \text{tri}\left(\frac{t}{T_0}\right)$	$\text{sinc}^2\left(\frac{\omega T_0}{2\pi}\right)$	$\sin(\omega t)$	$\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$

According to Fig. 6-83, for $f_s/B = 1.5$, the sampling rate should be $f_s = 3B = 3 \times 100 = 300$ Hz.

Per Eq. (6.158), the spectrum of the sampled signal is given by

$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - n f_s)$$

Spectrum $X_s(f)$ consists of the original spectrum, plus an infinite number of duplicates, shifted to the right by $n f_s$ for $n > 0$ and to the left by $|n f_s|$ for $n < 0$. All spectra are scaled by a multiplicative factor of f_s . Figure 6-86(b) displays the spectra for $n = 0$ and ± 1 . We note that the spectra do not overlap, which means that the original signal can be reconstructed by passing spectrum $X_s(f)$ through a lowpass filter with a spectral response that extends between -150 Hz and $+150$ Hz.

6-13.14 Practical Aspects of Sampling

So far, all our discussions of signal sampling assumed the availability of an ideal impulse train composed of impulses.

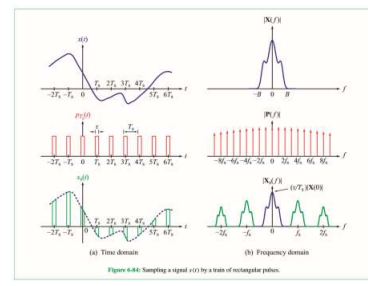
In practice, the sampling is performed by finite-duration pulses that may resemble rectangular or Gaussian waveforms. Figure 6-84 depicts the sampling operation for a signal $x(t)$, with the sampling generated by a train of rectangular pulses given by

$$p_T(t) = \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t - nT_s}{\tau}\right)$$

where T_s is the sampling interval and τ is the pulse width. From Table 5-4, the Fourier series representation of the pulse train is

$$p_T(t) = \frac{\tau}{T_s} + \sum_{n=1}^{\infty} \frac{2}{m\pi} \sin\left(\frac{m\pi\tau}{T_s}\right) \cos\left(\frac{2m\pi\tau}{T_s} t\right) \quad (6.166)$$

The signal sampled by the pulse train is

$$x_s(t) = x(t) p_T(t) = \frac{\tau}{T_s} x(t) + \sum_{n=1}^{\infty} \frac{2}{m\pi} x(t) \sin\left(\frac{m\pi\tau}{T_s}\right) \cos\left(\frac{2m\pi\tau}{T_s} t\right) \quad (6.167)$$


which can be cast in the form

$$x_s(t) = A_0 x(t) + A_1 x(t) \cos(2\pi f_s t) + A_2 x(t) \cos(4\pi f_s t) + \dots \quad (6.168)$$

with

$$A_0 = \frac{\tau}{T_s}, \quad A_1 = \frac{2}{\pi} \sin\left(\frac{\pi\tau}{T_s}\right), \quad A_2 = \frac{2}{\pi} \sin\left(\frac{2\pi\tau}{T_s}\right), \dots$$

The sequence given by Eq. (6.168) consists of a dc term, $A_0 x(t)$, and a sum of sinusoids at frequency f_s and its harmonics. The Fourier transform of the A_n term is $A_n X(f)$, where $X(f)$ is the transform of $x(t)$. It is represented by the central spectrum of $X_s(f)$ in Fig. 6-86(b). The cosine terms in Eq. (6.168) generate image spectra centered at $\pm f_s$ and its harmonics, but their amplitudes are modulated by the values of A_1, A_2 , etc.

Signal $x(t)$ can be reconstructed from $x_s(t)$ by passing through a filter that was close earlier with the ideal impulse sampling. The Shannon sampling requirement that f_s should be greater than $2B$ still holds.

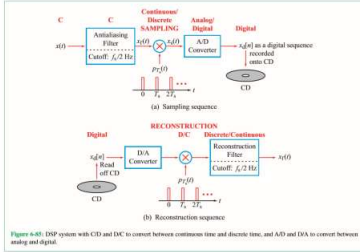


Figure 6-86: DSP system with CD and DAC to convert between continuous time and discrete time, and AD and DA to convert between analog and digital.

By way of an example, we show in Fig. 6-88 a block diagram of a typical DSP system that uses a compact disc (CD) for storage of digital data. The sampling part of the process starts with a continuous-time signal $x(t)$. After lowpass filtering the signal by $f_c/2$ by an anti-aliasing filter, the filtered signal $x_f(t)$ is sampled at a rate f_s and then converted into a digital sequence $x_d[n]$ that gets recorded onto the CD.

Reconstruction performs the reverse sequence, starting with $x_d[n]$ as read off of the CD and concluding in $x(t)$, the bandwidth version of the original signal.

Concept Question 6-19: Does sampling a signal at exactly the Nyquist rate guarantee that it can be reconstructed from its discrete samples? (See [Eq. \(6.1\)](#).)

Concept Question 6-20: What is signal aliasing? What causes it? How can it be avoided? (See [Eq. \(6.1\)](#).)

Concept Question 6-21: If brick-wall lowpass filters are used in connection with a signal bandwidth f_m and sampled at f_s , what should the filter's cutoff frequency be when used as (a) an anti-aliasing filter and (b) a reconstruction filter? (See [Eq. \(6.1\)](#).)

Exercise 6-18: What is the Nyquist sampling rate for a signal bandwidth of 5 kHz? (See [Eq. \(6.1\)](#).)

Exercise 6-19: A 500 Hz sinusoid is sampled at 800 Hz. No anti-alias filter is used. What is the frequency of the reconstructed sinusoid? (See [Eq. \(6.1\)](#).)