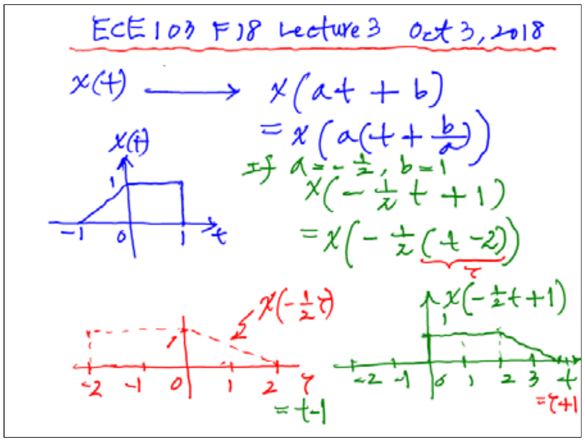
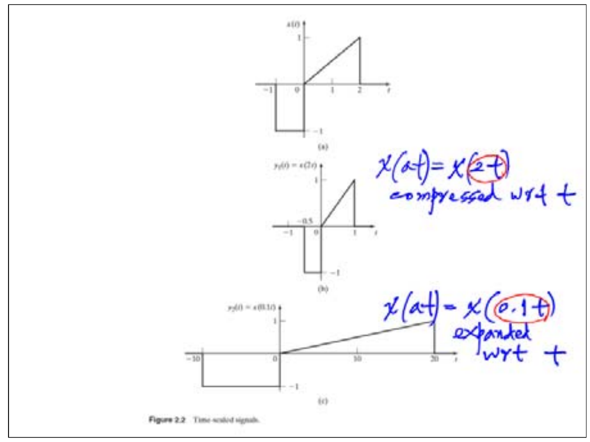
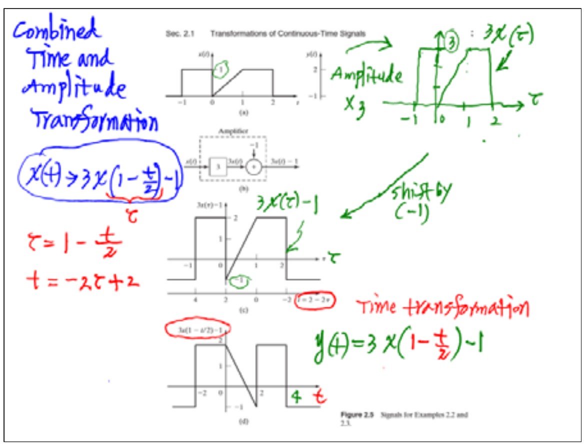
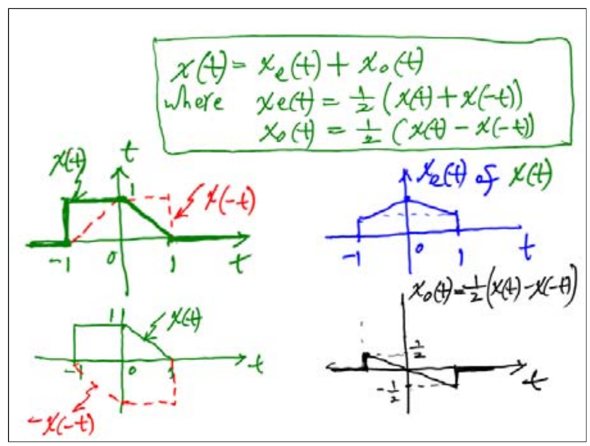
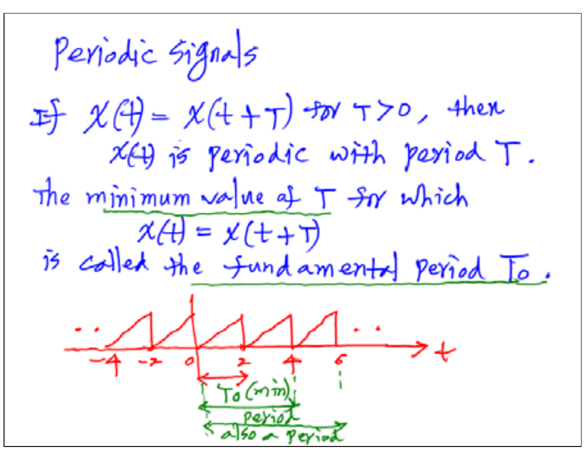
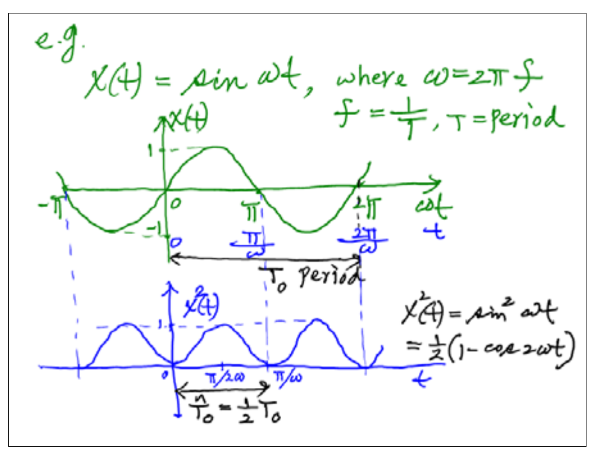

ECE 103 F18 Lecture 3 Oct 3, 2018
x(t) ⟶ x(at + b)
= x(a(t + b/a))
If a = -1/2, b = 1
x(-1/2 t + 1)
= x(-1/2 (t - 2))
x(-1/2 τ)
x(-1/2 t + 1)
= t - 1
= τ + 1

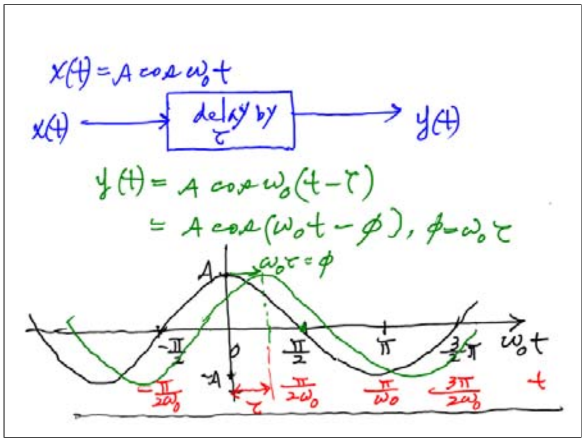

x(at) = x(2t)
compressed wrt t
x(at) = x(0.1t)
expanded wrt t
Figure 2.2 Time-scaled signals.

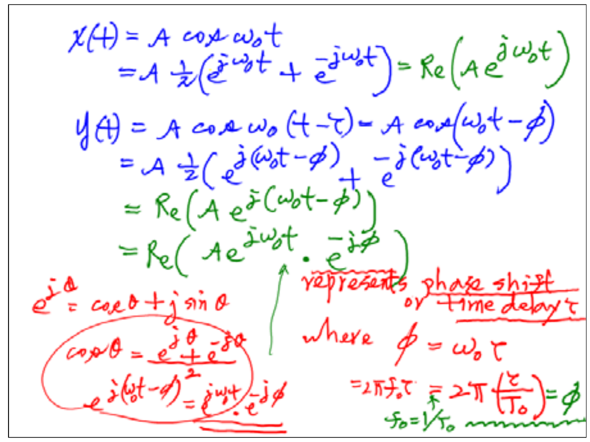

Combined Time and Amplitude Transformation
Sec. 2.1 Transformations of Continuous-Time Signals
Amplitude x3
3x(τ)
Amplifier
x(t) → 3x(1 - t/2) - 1
τ = 1 - t/2
t = -2τ + 2
3x(τ) - 1
shift by (-1)
Time transformation
y(t) = 3x(1 - t/2) - 1
Figure 2.3 Signals for Examples 2.2 and 2.3.

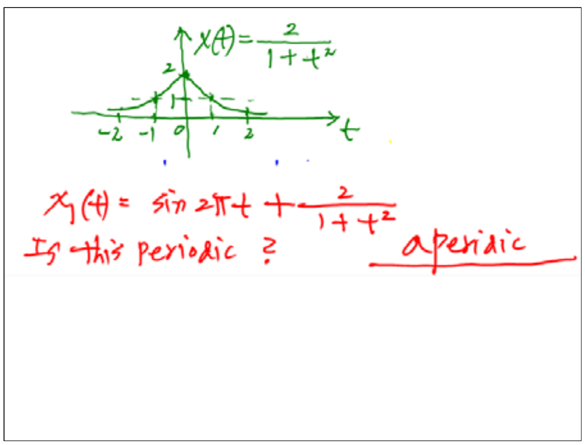

x(t) = x_e(t) + x_o(t)
where x_e(t) = 1/2 (x(t) + x(-t))
x_o(t) = 1/2 (x(t) - x(-t))
x_e(t) of x(t)
x_o(t) = 1/2 (x(t) - x(-t))
-x(-t)

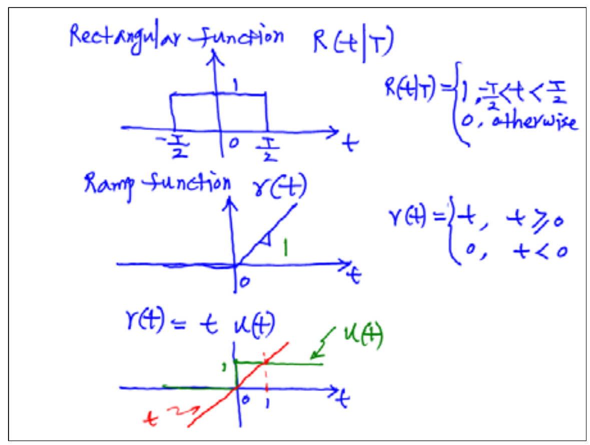

Periodic signals
If x(t) = x(t + T) for T > 0, then
x(t) is periodic with period T.
The minimum value of T for which
x(t) = x(t + T)
is called the fundamental period T0.
T0 (min) period
also a period

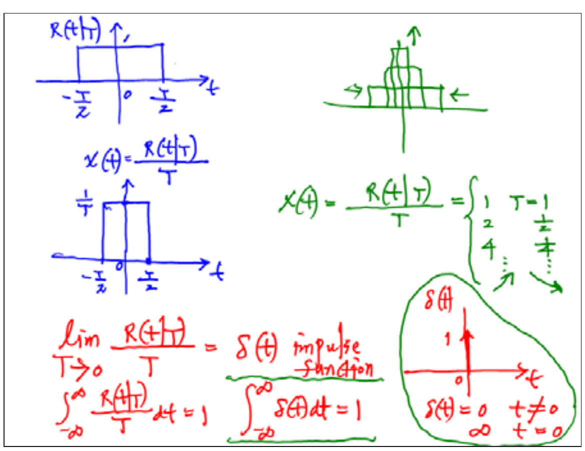

e.g.
x(t) = sin ωt, where ω = 2πf
f = 1/T, T = period
T0 period
x²(t) = sin² ωt
= 1/2 (1 - cos 2ωt)
T0' = 1/2 T0

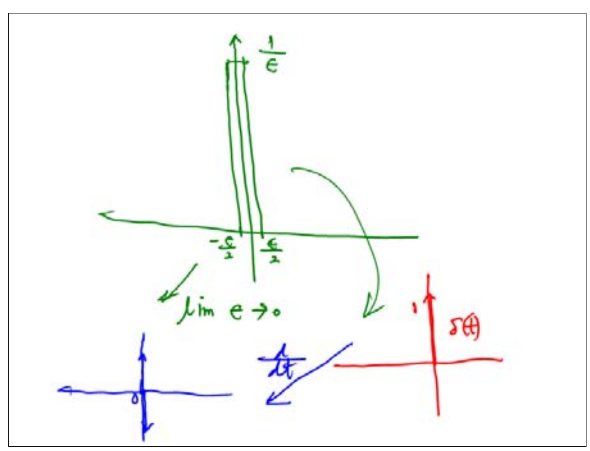

$x(t) = A\cos\omega_0 t$
delay by $\tau$
$x(t)$
$y(t)$
$y(t) = A\cos\omega_0(t-\tau)$
$= A\cos(\omega_0 t - \phi), \phi = \omega_0\tau$
$\omega_0\tau = \phi$
$\omega_0 t$
$t$

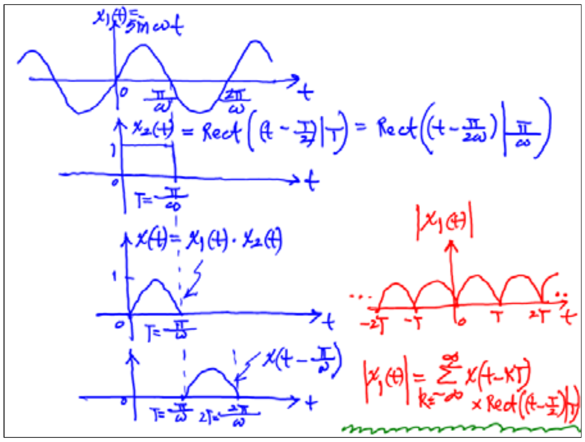

$x(t) = A\cos\omega_0 t$
$= A\frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t}) = Re(Ae^{j\omega_0 t})$
$y(t) = A\cos\omega_0(t-\tau) = A\cos(\omega_0 t - \phi)$
$= A\frac{1}{2}(e^{j(\omega_0 t-\phi)} + e^{-j(\omega_0 t-\phi)})$
$= Re(Ae^{j(\omega_0 t-\phi)})$
$= Re(Ae^{j\omega_0 t}\cdot e^{-j\phi})$
represents phase shift or time delay $\tau$
where $\phi = \omega_0\tau$
$= 2\pi f_0\tau = 2\pi(\frac{\tau}{T_0}) = \phi$
$f_0 = 1/T_0$
$e^{j\theta} = \cos\theta + j\sin\theta$
$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$
$e^{j(\omega_0 t-\phi)} = e^{j\omega_0 t}\cdot e^{-j\phi}$

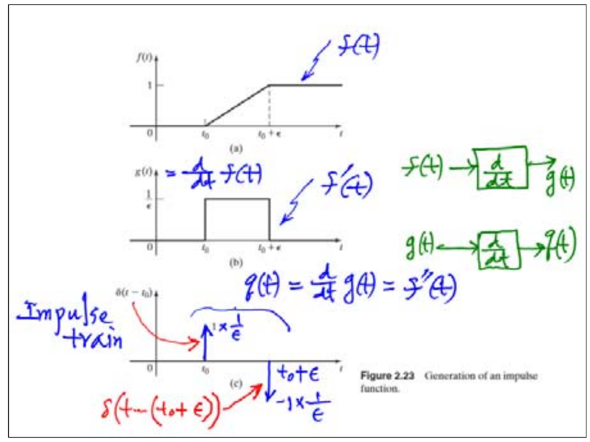

$x(t) = \frac{2}{1+t^2}$
$x_1(t) = \sin 2\pi t + \frac{2}{1+t^2}$
Is this periodic ?
aperiodic

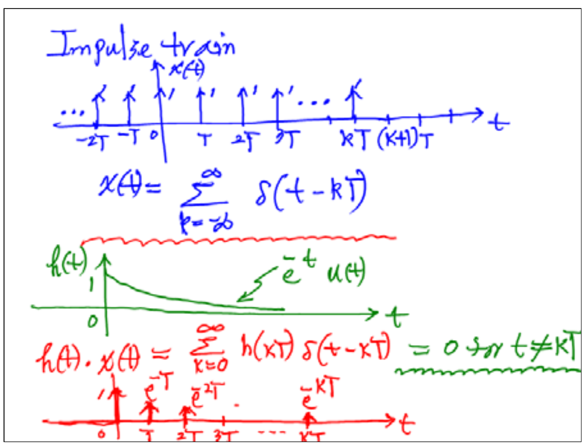

Rectangular function $R(t|T)$
$R(t|T) = \begin{cases} 1, & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$
Ramp function $r(t)$
$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$
$r(t) = t\,u(t)$
$u(t)$

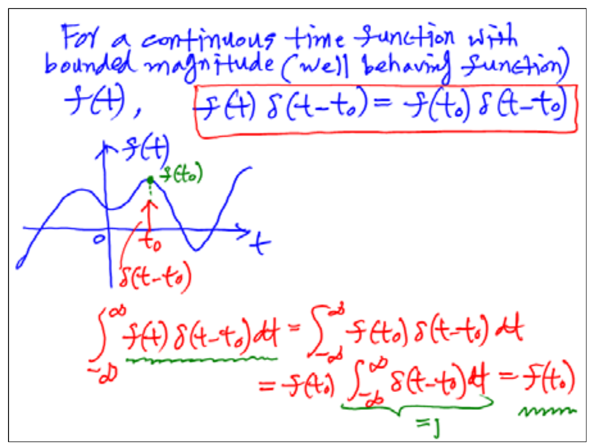

$R(t|T)$
$x(t) = \frac{R(t|T)}{T}$
$x(t) = \frac{R(t|T)}{T} = \begin{cases} 1 & T=1 \\ 2 & \frac{1}{2} \\ 4 & \frac{1}{4} \end{cases}$
$\lim_{T\to 0}\frac{R(t|T)}{T} = \delta(t)$ impulse function
$\int_{-\infty}^{\infty}\frac{R(t|T)}{T}dt = 1$
$\int_{-\infty}^{\infty}\delta(t)dt = 1$
$\delta(t) = 0 \quad t \neq 0$
$\infty \quad t = 0$


$\frac{1}{\epsilon}$
$-\frac{\epsilon}{2}$ $\frac{\epsilon}{2}$
$\lim \epsilon \to 0$
$\delta(t)$
$\frac{d}{dt}$



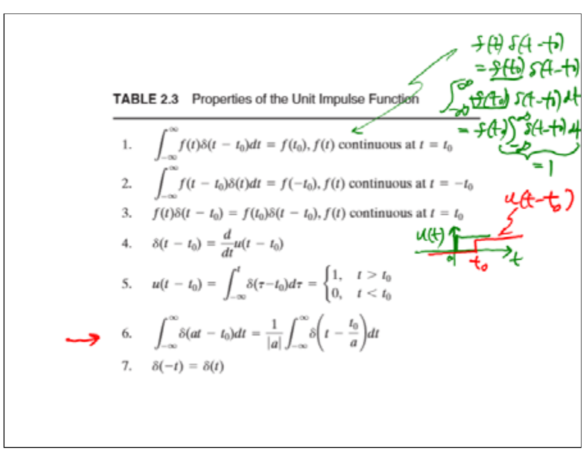
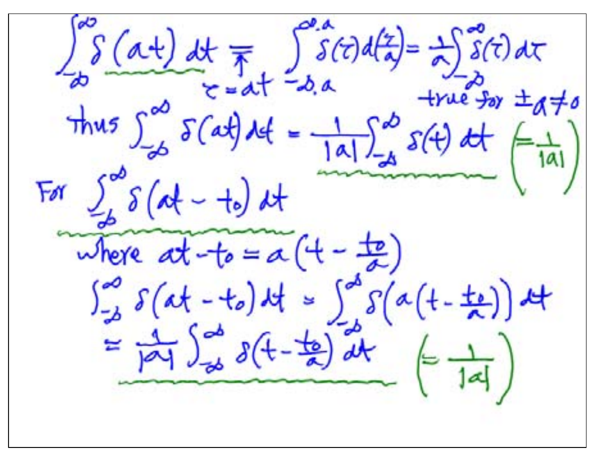



TABLE 2.3 Properties of the Unit Impulse Function

1. $\int_{-\infty}^{\infty} f(t)\delta(t - t_0)dt = f(t_0)$, $f(t)$ continuous at $t = t_0$
2. $\int_{-\infty}^{\infty} f(t - t_0)\delta(t)dt = f(-t_0)$, $f(t)$ continuous at $t = -t_0$
3. $f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$, $f(t)$ continuous at $t = t_0$
4. $\delta(t - t_0) = \frac{d}{dt}u(t - t_0)$
5. $u(t - t_0) = \int_{-\infty}^{t} \delta(\tau - t_0)d\tau = \begin{cases} 1, & t > t_0 \\ 0, & t < t_0 \end{cases}$
6. $\int_{-\infty}^{\infty} \delta(at - t_0)dt = \frac{1}{|a|}\int_{-\infty}^{\infty} \delta\left(t - \frac{t_0}{a}\right)dt$
7. $\delta(-t) = \delta(t)$





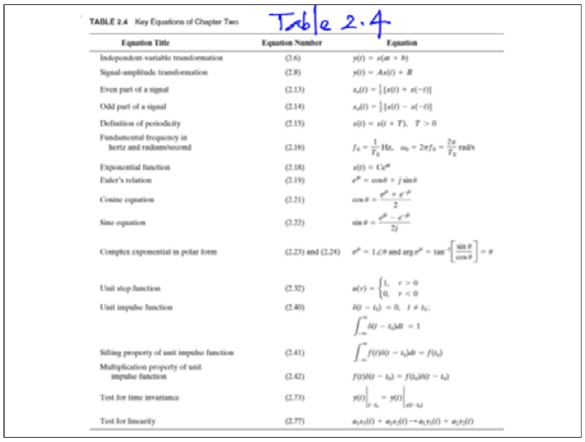
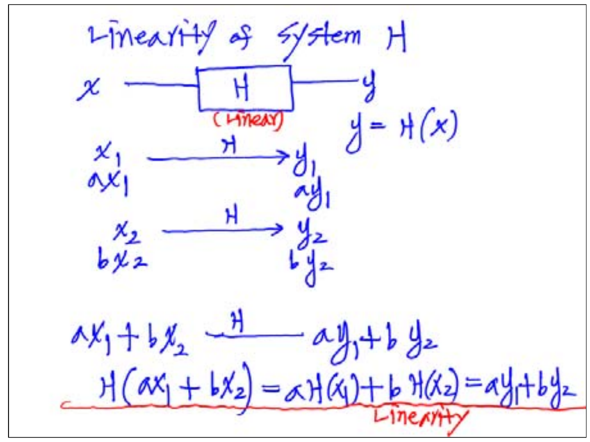
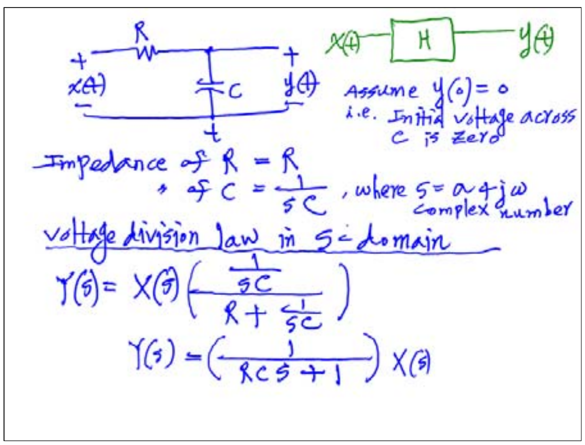


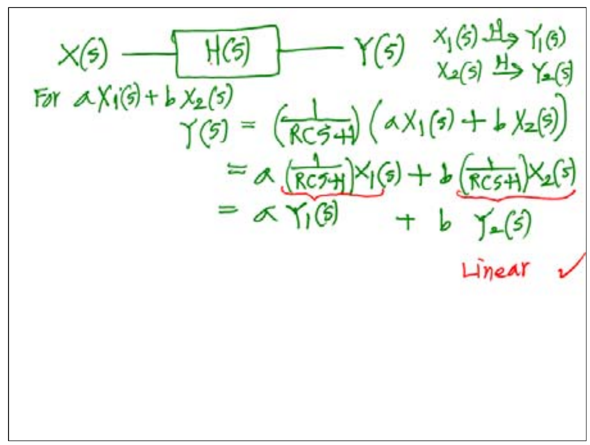
Table 2.4

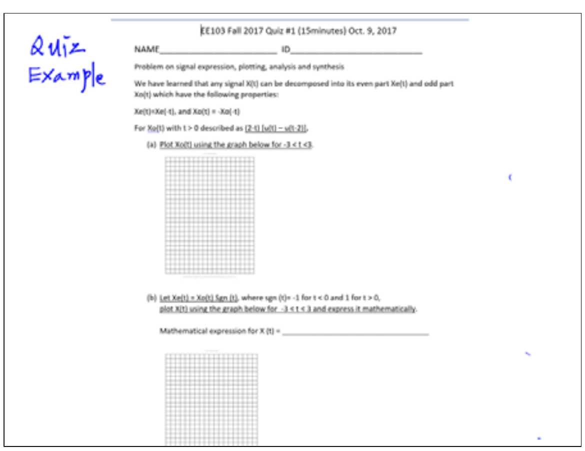
TABLE 2.4 Key Equations of Chapter Two

| Equation Title | Equation Number | Equation |
| --- | --- | --- |
| Independent-variable transformation | (2.6) | $y(t) = x(at + b)$ |
| Signal-amplitude transformation | (2.8) | $y(t) = Ax(t) + B$ |
| Even part of a signal | (2.13) | $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ |
| Odd part of a signal | (2.14) | $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$ |
| Definition of periodicity | (2.15) | $x(t) = x(t + T),\ T > 0$ |
| Fundamental frequency in hertz and radians/second | (2.16) | $f_0 = \frac{1}{T_0}$ Hz, $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$ rad/s |
| Exponential function | (2.18) | $x(t) = Ce^{at}$ |
| Euler's relation | (2.19) | $e^{j\theta} = \cos\theta + j\sin\theta$ |
| Cosine equation | (2.21) | $\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$ |
| Sine equation | (2.22) | $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$ |
| Complex exponential in polar form | (2.23) and (2.24) | $e^{j\theta} = 1\angle\theta$ and $\arg e^{j\theta} = \tan^{-1}\left[\frac{\sin\theta}{\cos\theta}\right] = \theta$ |
| Unit step function | (2.32) | $u(\tau) = \begin{cases} 1, & \tau > 0 \\ 0, & \tau < 0 \end{cases}$ |
| Unit impulse function | (2.40) | $\delta(t - t_0) = 0,\ t \neq t_0;$ $\int_{-\infty}^{\infty} \delta(t - t_0)dt = 1$ |
| Sifting property of unit impulse function | (2.41) | $\int_{-\infty}^{\infty} f(t)\delta(t - t_0)dt = f(t_0)$ |
| Multiplication property of unit impulse function | (2.42) | $f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$ |
| Test for time invariance | (2.73) | $y(t)\Big\|_{t-t_0} = y(t)\Big\|_{x(t-t_0)}$ |
| Test for linearity | (2.77) | $a_1x_1(t) + a_2x_2(t) \to a_1y_1(t) + a_2y_2(t)$ |





Quiz Example

EE103 Fall 2017 Quiz #1 (15minutes) Oct. 9, 2017

NAME________________________ ID__________________________

Problem on signal expression, plotting, analysis and synthesis

We have learned that any signal X(t) can be decomposed into its even part Xe(t) and odd part Xo(t) which have the following properties:

Xe(t)=Xe(-t), and Xo(t) = -Xo(-t)

For Xo(t) with t > 0 described as (2-t) [u(t) – u(t-2)].

(a) Plot Xo(t) using the graph below for -3 < t <3.

(b) Let Xe(t) = Xo(t) Sgn (t), where sgn (t)= -1 for t < 0 and 1 for t > 0, plot X(t) using the graph below for -3 < t < 3 and express it mathematically.

Mathematical expression for X (t) = ________________________________