ECE 103 F18 Lecture Oct 5, 2018

- Reminder Quiz #1 on Monday Oct 8 (3:30-3:45)

DRC test in Room E2-234

Quiz #1 covers up to Lect #4

HW #1

Table 2.4

TABLE 2.4 Key Equations of Chapter Two

Equation Title	Equation Number	Equation
Independent-variable transformation	(2.6)	$y(t) = x(at + b)$ $\leftrightarrow t$
Signal-amplitude transformation	(2.8)	$y(t) = Ax(t) + B$
Even part of a signal	(2.13)	$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$
Odd part of a signal	(2.14)	$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$
Definition of periodicity	(2.15)	$x(t) = x(t + T),\ T > 0$
Fundamental frequency in hertz and radians/second	(2.16)	$f_0 = \frac{1}{T_0}$ Hz, $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$ rad/s
Exponential function	(2.18)	$x(t) = Ce^{at}$
Euler's relation	(2.19)	$e^{j\theta} = \cos\theta + j\sin\theta$
Cosine equation	(2.21)	✓ $\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$
Sine equation	(2.22)	✓ $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$
Complex exponential in polar form	(2.23) and (2.24)	$e^{j\theta} = 1\angle\theta$ and $\arg e^{j\theta} = \tan^{-1}\left[\frac{\sin\theta}{\cos\theta}\right] = \theta$
Unit step function	(2.32)	$u(\tau) = \begin{cases} 1, & \tau > 0 \\ 0, & \tau < 0 \end{cases}$
Unit impulse function	(2.40)	$\delta(t - t_0) = 0,\ t \neq t_0;$ $\int_{-\infty}^{\infty} \delta(t - t_0)dt = 1$
Sifting property of unit impulse function	(2.41)	$\int_{-\infty}^{\infty} f(t)\delta(t - t_0)dt = f(t_0)$
Multiplication property of unit impulse function	(2.42)	$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$
Test for time invariance	(2.73)	$y(t)\big\vert_{t-t_0} = y(t)\big\vert_{x(t-t_0)}$
Test for linearity	(2.77)	$a_1x_1(t) + a_2x_2(t) \rightarrow a_1y_1(t) + a_2y_2(t)$

[Q] Is $x(t) = 5\sin(15t - 60^\circ) + 2\sin 7t$ periodic?

Test $x(t) \overset{?}{=} x(t+T)$ for $T > 0$?

$x(t+T) = 5\sin\left(15(t+T) - 60^\circ\right) + 2\sin 7(t+T)$

$= 5\sin\left((15t - 60^\circ) + 15T\right) + 2\sin(7t + 7T)$

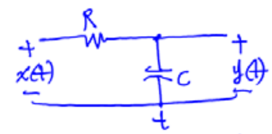
For $T = 0,\ \boxed{2\pi},\ 4\pi,$ (even number $\times 2\pi$)

$x(t) = x(t+T)$ periodic.

Hint $\sin(\alpha + 2\pi n) = \sin\alpha\cos 2\pi n + \cos\alpha\sin 2\pi n$

n integer $= \sin\alpha$

where $\alpha = (15t - 60^\circ),\ 7t$



Assume $y(0_-) = 0$

i.e. Initial voltage across C is 0

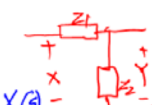
Impedance of $R = R$

of $C = \frac{1}{sC}$, where $s = \sigma + j\omega$ complex number

voltage division law in s-domain

$$Y(s) = X(s)\left(\frac{\frac{1}{sC}}{R + \frac{1}{sC}}\right)$$

$$= \left(\frac{1}{RCs + 1}\right)X(s)$$



$Y = \frac{Z_2}{Z_1 + Z_2}X$

(I) when $x(t) = \delta(t)$ impulse input

$$X(s) = \mathcal{L}[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)e^{-st}dt = \int_{-\infty}^{\infty} \delta(t)e^{-s(0)}dt$$

Laplace transform (chapter 7)

$$= e^{-s(0)}\int_{-\infty}^{\infty} \delta(t)dt$$

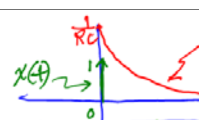
$$= e^{0}\cdot 1 = 1 = \Delta(s)$$

$$Y(s) = \frac{1}{RCs + 1}\Delta(s) = \frac{1}{RCs + 1}\cdot 1$$

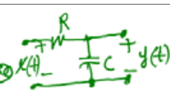
$$= \frac{1}{RC}\left(\frac{1}{s + \frac{1}{RC}}\right)$$

$$y(t) = \mathcal{L}^{-1}Y(s) = \mathcal{L}^{-1}\frac{1}{RC}\left(\frac{1}{s + \frac{1}{RC}}\right) = \frac{1}{RC}e^{-\frac{1}{RC}t}$$

$\mathcal{L}^{-1}\left(\frac{b}{s+a}\right) = be^{-at}$



$\frac{1}{RC}e^{-\frac{1}{RC}t} = y(t)$ (Impulse response)



$\boxed{x(t) = \delta(t) \longrightarrow y(t) = h(t)}$

(II) when $x(t) = u(t)$

$$\mathcal{L}[x(t)] = \int_{-\infty}^{\infty} u(t)e^{-st}dt = \int_{0}^{\infty} 1\, e^{-st}dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^{\infty} = \frac{e^{-s(\infty)}}{-s} + \frac{e^{-s(0)}}{s} = \frac{1}{s}$$

$$Y(s) = \frac{1}{RCs + 1}\cdot\frac{1}{s} = \frac{1}{RC}\left(\frac{1}{s + \frac{1}{RC}}\right)\frac{1}{s}$$

$$= \frac{1}{RC}\left(\frac{A}{s + \frac{1}{RC}} + \frac{B}{s}\right)$$ (next page)

$$= \frac{1}{RC}\left(\frac{A}{s+\frac{1}{RC}} + \frac{B}{s}\right) \left[= \frac{1}{RC}\left(\frac{1}{s+\frac{1}{RC}}\frac{1}{s}\right)\right]$$

calculation of A, B

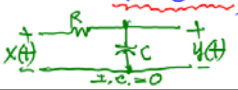
$$A = \left(\frac{1}{s+\frac{1}{RC}}\frac{1}{s}\right) \times \left(s+\frac{1}{RC}\right)\Big|_{s=-\frac{1}{RC}} = -RC$$

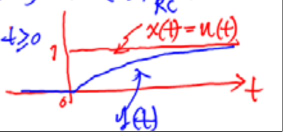
$$B = \left(\frac{1}{s+\frac{1}{RC}}\frac{1}{s}\right) \times s\Big|_{s=0} = \frac{1}{1/RC} = RC$$

Thus $Y(s) = \frac{1}{RC}\left(\frac{-RC}{s+\frac{1}{RC}} + \frac{RC}{s}\right) = \frac{1}{s} - \frac{1}{s+\frac{1}{RC}}$

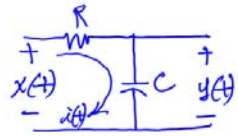
$$y(t) = \mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1}\left(\frac{1}{s}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+\frac{1}{RC}}\right)$$

$$= 1 - e^{-\frac{1}{RC}t}, \quad t \geq 0$$





Revisit of RC circuit in time domain



$$y(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = \frac{1}{C}\int_{-\infty}^{0} i(\tau)d\tau + \frac{1}{C}\int_{0}^{t} i(\tau)d\tau$$

$$i(t) = \frac{1}{R}(x(t) - y(t))$$

Alternatively $\quad y(t) = x(t) - R\,i(t) \quad (1)$

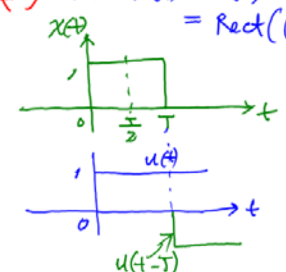
$$i(t) = C\frac{dy(t)}{dt} \quad (2)$$

$$y(t) = x(t) - RC\frac{dy(t)}{dt}$$

$\boxed{RC\frac{dy}{dt} + y = x}$ first-order d.e.

(III) For $x(t) = u(t) - u(t-T)$

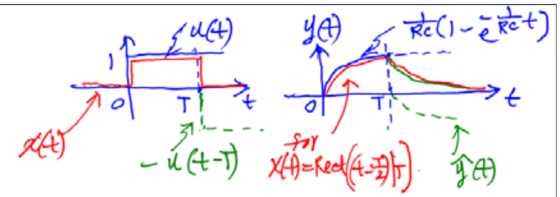
$= \text{Rect}((t-\frac{T}{2})|T)$?



Linear, time-invariant system

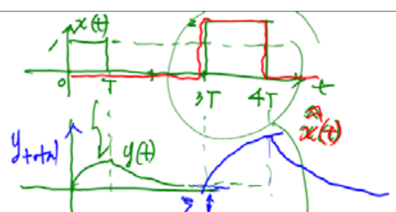
(R, C do not change with t, always same)

$\boxed{\text{Can we find } y(t) \text{ ?}}$

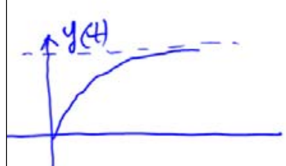


$$\hat{y}(t) = \begin{cases} 0 & t < T \\ -u(t-T)\frac{1}{RC}\left(1-e^{-\frac{1}{RC}(t-T)}\right) \end{cases}$$

$$y(t) = u(t)\frac{1}{RC}\left(1-e^{-\frac{1}{RC}t}\right) - u(t-T)\frac{1}{RC}\left(1-e^{-\frac{1}{RC}(t-T)}\right)$$



$\hat{y}(t) = y(t)$ shifted by $3T$
scaled by $\times 2$



$1 - e^{-\frac{t}{RC}} = 1 - e^{-t/\tau}$

time constant $\tau = RC$

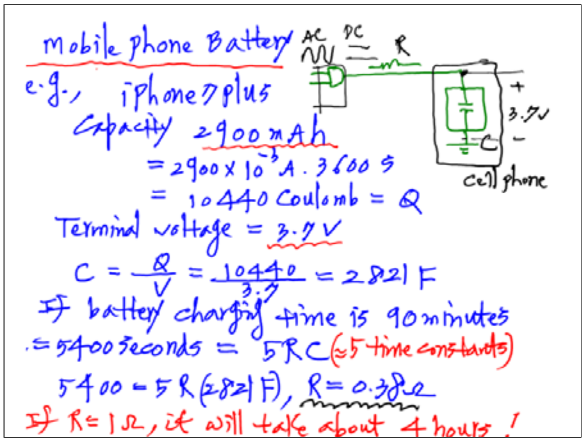
when

$t = 5RC$

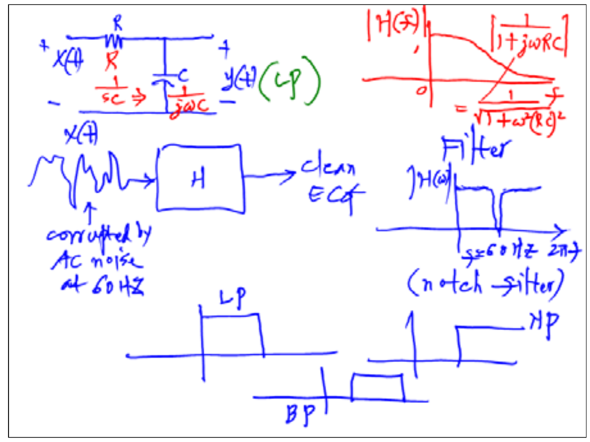
$1 - e^{-5} = 1 - 0.0067$

$\approx 1 \quad 0.9933$

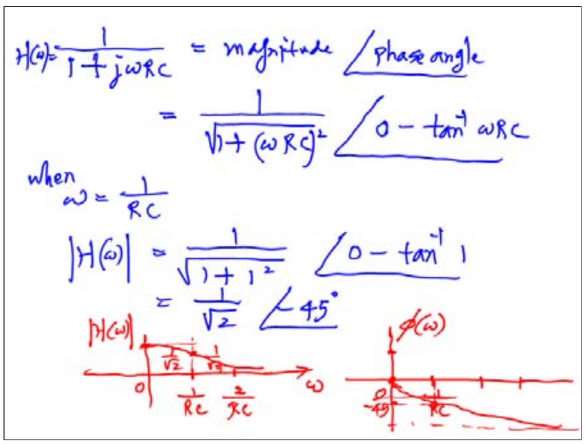
$y(t) \approx 1$ at $t = 5RC$



Mobile phone Battery
AC
DC
R
+
3.7V
−
C
cell phone
e.g., iPhone 7 plus
Capacity 2900 mAh
= 2900 x 10^-3 A . 3600 s
= 10440 Coulomb = Q
Terminal voltage = 3.7 V
C = Q/V = 10440/3.7 = 2821 F
If battery charging time is 90 minutes
≈ 5400 seconds ≈ 5RC (≈ 5 time constants)
5400 = 5 R (2821 F), R = 0.38 Ω
If R = 1 Ω, it will take about 4 hours !



R
x(t)
R
1/sC → 1/jωC
C
y(t) (LP)
|H(f)|
|1/(1+jωRC)|
0
f
= 1/√(1+ω²(RC)²)
x(t)
H
clean ECG
corrupted by AC noise at 60 HZ
Filter
|H(ω)|
f = 60 Hz
2πf
(notch filter)
LP
HP
BP



H(ω) = 1/(1 + jωRC) = magnitude ∠phase angle
= 1/√(1+(ωRC)²) ∠0 − tan⁻¹ ωRC
when ω = 1/RC
|H(ω)| = 1/√(1+1²) ∠0 − tan⁻¹ 1
= 1/√2 ∠−45°
|H(ω)|
1/√2
0
1/RC
2/RC
ω
φ(ω)
−45