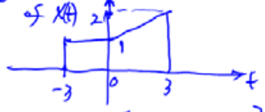
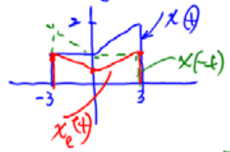
ECE103 Lecture #5 Oct 8, 2018

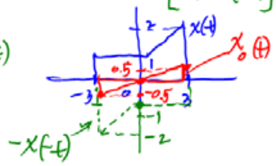
[2] 5th ed. Prob. 2-5(a) plot even & odd parts of $x(t)$



$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$



$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$



[4] $x(t) = 3\cos 2t + \sin 2t \quad (1)$

$= A\cos(\omega_0 t + \phi)$

$\cos 2t \underset{\text{Euler's formula}}{=} \dfrac{e^{j2t} + e^{-j2t}}{2}, \quad \sin 2t = \dfrac{e^{j2t} - e^{-j2t}}{2j}$

$(1) \rightarrow 3\,\dfrac{e^{j2t} + e^{-j2t}}{2} + \dfrac{e^{j2t} - e^{-j2t}}{2j}$

$= e^{j2t}\left(\frac{3}{2} + \frac{1}{2j}\right) + e^{-j2t}\left(\frac{3}{2} - \frac{1}{2j}\right) \quad (2)$

$\frac{3}{2} + \frac{1}{2j} = \frac{1}{2}(3 - j) = \frac{1}{2}\sqrt{10}\, e^{-j\tan^{-1}\frac{1}{3}} \quad (3)$

$\frac{3}{2} - \frac{1}{2j} = \frac{1}{2}(3 + j) = \frac{1}{2}\sqrt{10}\, e^{+j\tan^{-1}\frac{1}{3}} \quad (4)$

$(3),(4) \rightarrow (2) \Rightarrow \frac{1}{2}\sqrt{10}\left[e^{j(2t - \tan^{-1}\frac{1}{3})} + e^{-j(2t - \tan^{-1}\frac{1}{3})}\right]$

$= \sqrt{10}\cos(2t - \tan^{-1}\frac{1}{3})$ ans

Convolution

$y(t) = \underline{x(t) * h(t)}$

$= \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau$

$\underset{\uparrow}{=} \int_{-\infty}^{t} x(t-\tau)\, h(\tau)\, d\tau = \underline{h(t) * x(t)}$

$(t-\tau) \leftarrow \tau$

$\tau \leftarrow t-\tau$

(commutative)

e.g.) $x(t) = u(t), \; h(t) = e^{-t}u(t)$

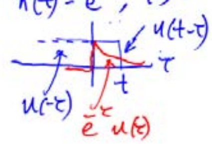
$x(t) * h(t) = \int_{-\infty}^{t} u(\tau)\, e^{-(t-\tau)}\, d\tau = \int_{0}^{t} e^{-t} e^{\tau}\, d\tau = e^{-t}\left[e^{\tau}\right]_0^t$

$= e^{-t}(e^{t} - 1) = \underline{(1 - e^{-t}), \; t > 0}$

$h(t) * x(t) = \int_0^t e^{-\tau} \cdot u(t-\tau)\, d\tau$

$h(t) = e^{-t}, \; t > 0$

$= -e^{-\tau}\Big|_0^t = -e^{-t} + 1$

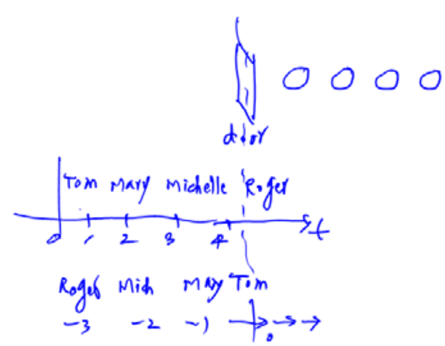


$= 1 - e^{-t} = x(t) * h(t)$

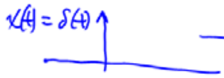
$\boxed{\text{Convolution}}$ (def. by Webster dictionary)

A twisting, coiling, or winding together

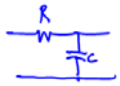
Any of the irregular folds or ridges on the surface of the brain (inside the skull)



$x(t)=\delta(t) \to \boxed{H} \to y(t)=h(t)$ impulse response



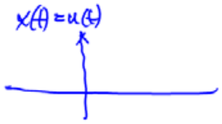
e.g. H



$h(t)=\frac{1}{RC}e^{-\frac{1}{RC}t}$, $t>0$

when $RC=1$, $h(t)=e^{-t}$

$x(t)=u(t)$



$\xrightarrow{H} y(t)$

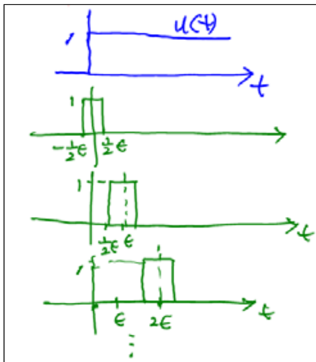
$\delta(t) \to \boxed{\int} \xrightarrow{u(t)} \boxed{H} \to y(t)$ for $x(t)=u(t)$

integration

$\delta(t) \to \boxed{H} \to \boxed{\int} \to$

$h(t)=\frac{1}{RC}e^{-\frac{1}{RC}t}$

$$\int_0^t \frac{1}{RC}e^{-\frac{1}{RC}\tau}d\tau = 1-e^{-\frac{1}{RC}t},\ t>0$$



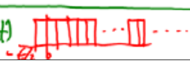
$x_1(t)=\mathrm{Rect}(t|\epsilon)$

$x_2(t)=\mathrm{Rect}((t-\epsilon)|\epsilon)$

$x_3(t)=\mathrm{Rect}((t-2\epsilon)|\epsilon)$

$x_k(t)=\mathrm{Rect}((t-k\epsilon)|\epsilon)$

$\sum_k x_k(t)$



$x(t)=u(t) \to \boxed{H} \to y(t)=H(x(t))$

$$x(t)=\lim_{\epsilon\to 0}\sum_{k=0}^{\infty}\mathrm{Rect}((t-k\epsilon)|\epsilon) = u(t)$$

$$y(t)=H(x(t)) = H\lim_{\epsilon\to 0}\sum_{k=0}^{\infty}\mathrm{Rect}((t-k\epsilon)|\epsilon)$$

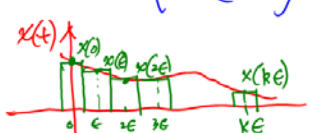
$$= \lim_{\epsilon\to 0}\sum_{k=0}^{\infty} H\,\epsilon\,\frac{\mathrm{Rect}((t-k\epsilon)|\epsilon)}{\epsilon}$$

$$= \sum_{k=0}^{\infty}\epsilon\ \underbrace{H\underbrace{\lim_{\epsilon\to 0}\left(\frac{\mathrm{Rect}((t-k\epsilon)|\epsilon)}{\epsilon}\right)}_{=\delta(t-k\epsilon)}}_{\frac{1}{RC}e^{-\frac{1}{RC}(t-k\epsilon)}u(t-k\epsilon)}$$

If we set $k\epsilon=\tau$, $\epsilon=d\tau$

$$\to \int_0^t \frac{1}{RC}e^{-(t-\tau)}d\tau$$

$$=\left(1-e^{-\frac{1}{RC}t}\right)u(t) \quad t>0$$



$t>0$

$$H\,x(t)=\sum_{k=0}^{\infty}\lim_{\epsilon\to 0}x(k\epsilon)\,\epsilon\,H\,\frac{\mathrm{Rect}((t-k\epsilon)|\epsilon)}{\epsilon}$$

$$=\int_0^{\infty}x(\tau)\,h(t-\tau)\,d\tau$$