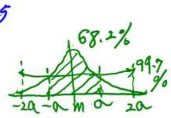
ECE 103 Lecture 8, Oct. 15, 2018

1. Quiz # 2 today put FULL name ~~not~~ just first name

2. HW # 3 assigned today

3. Start the topic of Fourier Series

4. Quiz 1 74 took Average = 5.69 σ = 2.33 68% between 3.3 and 8.0 mean

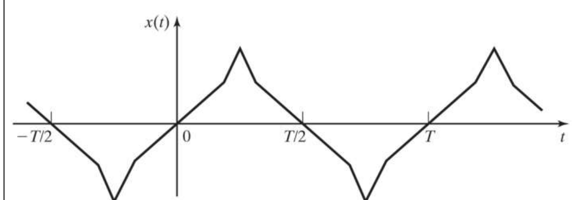


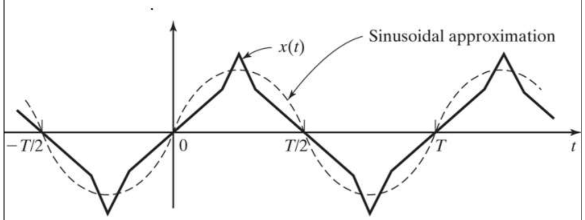
Joseph Fourier

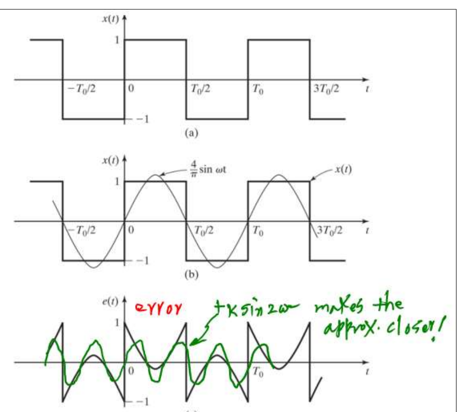
Jean-Baptiste Joseph Fourier

Jean Baptiste Joseph Fourier (born March 21, 1768, in Auxerre, Bourgogne, France; died May 16, 1830.) A mathematician known also as an Egyptologist and administrator, he exerted strong influence on mathematical physics through his *Théorie analytique de la chaleur* (1822; *The Analytical Theory of Heat*). He showed how the conduction of heat in solid bodies may be analyzed in terms of infinite mathematical series now called by his name, the Fourier series. Far transcending the particular subject of heat conduction, his work stimulated research in mathematical physics.

Reference: "Fourier, Joseph, Baron." Encyclopedia Britannica, 2007. Encyclopedia Britannica Online, January 6, 2007: http://www.britannica.com/eb/article-9035044







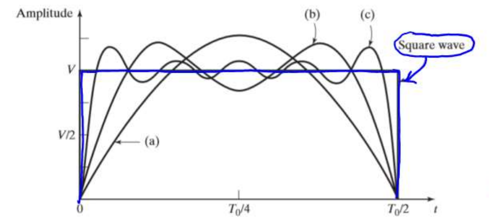


Figure 4.15 Truncated sums for a half-cycle of a square wave.

$\sum b_K \sin K\omega_0 t, \quad 0 < t < T_0/2$

Chapter 4 Fourier Series

TABLE 4.2 Forms of the Fourier Series

Name	Equation
Exponential	$\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}; \quad C_k = \lvert C_k\rvert e^{j\theta_k}, C_{-k} = C_k^*$
Combined trigonometric	$C_0 + \sum_{k=1}^{\infty} 2\lvert C_k\rvert \cos(k\omega_0 t + \theta_k)$
Trigonometric	$A_0 + \sum_{k=1}^{\infty}(A_k \cos k\omega_0 t + B_k \sin k\omega_0 t)$ $2C_k = A_k - jB_k, C_0 = A_0$ (*)
Coefficients	$C_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-jk\omega_0 t}\,dt$

From (*) $2C_K = A_K - jB_K$

and $2C_K^* = A_K + jB_K = C_{-K}$

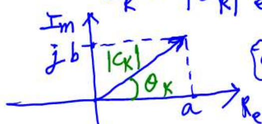
$2(C_K + C_K^*) = 2A_K$ $\qquad 2(C_K - C_K^*) = -2jB_K$

$A_K = C_K + C_{-K}$ $\qquad B_K = \frac{1}{-j}(C_K - C_K^*) = +j(C_K - C_{-K}) = B_K$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \qquad (1)$$

C_K is a complex number

$$C_k = |C_k| e^{j\theta_K} \qquad (2)$$



$\boxed{C_k = a + jb}$

$\theta_K = \tan^{-1}\frac{b}{a}$

From (1) & (2),

$$x(t) = \sum_{k=-\infty}^{\infty} |C_K| e^{j(k\omega_0 t + \theta_K)} \qquad (3)$$

$$= \underbrace{|C_0| e^{j(\theta_0)}}_{= C_0} + \sum_{k=1}^{\infty}\left(C_{-k} e^{-jk\omega_0 t} + C_{+k} e^{+jk\omega_0 t}\right) \qquad (4)$$

$$\text{where } C_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-jk\omega_0 t}\,dt \qquad (5)$$

$$C_{-k} = \frac{1}{T_0}\int_{T_0} x(t) e^{-j(-k)\omega_0 t}\,dt \qquad (6)$$

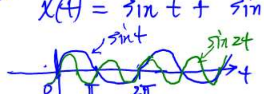
complex conjugate

$$C_K^* = \frac{1}{T_0}\int_{T_0} x(t) e^{+jk\omega_0 t} = C_{-k} \qquad (7)$$

$$|C_k| = |C_{-K}| \qquad (8)$$

$$\theta_{-k} = -\theta_k \qquad (9)$$

Example $x(t) = \sin t + \sin 2t$, $T_0 = 2\pi$, $\omega_0 = 1$



$$x(t) = \frac{e^{jt} - e^{-jt}}{2j} + \frac{e^{j2t} - e^{-j2t}}{2j}$$

$$= \frac{-1}{2j} e^{-j2t} + \frac{-1}{2j} e^{-jt} + \frac{e^{jt}}{2j} + \frac{e^{j2t}}{2j}$$

$$= \sum_{k=-2}^{2} C_k e^{jk\omega_0 t}$$

$C_0 = 0$

$C_1 = \frac{1}{2j}$, $C_{-1} = -\frac{1}{2j} = -C_1$

$C_2 = \frac{1}{2j}$ $C_{-2} = -\frac{1}{2j} = -C_2$

$$C_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-jk\omega_0 t}\,dt$$

where $T_0 = 2\pi$, $\omega_0 = 1$

$$C_1 = \frac{1}{2\pi}\int_{2\pi} (\sin t + \sin 2t) e^{-jt}\,dt$$

$$= \frac{1}{2\pi}\int_{2\pi}\left(\frac{e^{jt} - e^{-jt}}{2j} + \frac{e^{j2t} - e^{-j2t}}{2j}\right) e^{-jt}\,dt$$

$$= \frac{1}{2\pi}\int_{2\pi}\left(\frac{1 - e^{-j2t}}{2j} + \frac{e^{jt} - e^{-j3t}}{2j}\right) dt$$

$$= \frac{1}{2\pi}\int_{2\pi} \frac{1}{2j}\left(1 - e^{-j2t} + e^{jt} - e^{-j3t}\right) dt$$

$$= \frac{1}{2\pi}\frac{1}{2j} 2\pi + 0 + 0 + 0$$

$$= \frac{1}{2j}$$