[1]. (20 pts) For \( x(t) = \sin(t) \ u(t) \), where \( u(t) = 1 \) for \( t \geq 0 \), and 0 for \( t<0 \).

(a). (8 pts) Plot its even part \( x_e(t) \) in the graph below.

\[
x_e(t) = \frac{\sin(t \ u(t)) + \sin(t \ u(-t))}{2}
\]

(b). (12 pts) Plot the even odd part \( x_o(t) \) in the top graph below and then show that \( x(t) = x_e(t) + x_o(t) \) in the bottom-most graph.

\[
x_o(t) = \frac{x(t) - x(-t)}{2}
\]
[2]. (30 pts) Consider a system with its \( x(t) \)-to-\( y(t) \) relationship characterized by \( h(t) \).

![Diagram showing \( x(t) \) and \( y(t) \)]

(a). (15 pts) When \( h(t) = \exp(-2t) u(t-2) \) and \( x(t) = [u(t-1) - u(t-3)] \), find \( y(t) \) at \( t=4 \) by using a convolution integral.

\[
y(t=4) = \int_{0}^{3} h(t) x(t-t) \, dt \\
= \int_{2}^{3} \frac{e^{-2t}}{t} \, dt \\
= -\frac{e^{-4} - e^{-6}}{2}
\]

(b). (15 pts) Let \( H(\omega) = 1/(2+j\omega) \), where \( \omega = 2\pi f \), find \( y(t) \) for \( x(t) = 2 \cos(2t + 30^\circ) \).

You can express \( y(t) \) as a sinusoidal function with a phase angle in the form of \( \arctan \), with specific numerical values of a, b. For reference, \( \arctan x \) values for \( x = 0, 1, 2 \) are 0, 45 and 63.4 degrees, respectively.

\[
x(A) = 2 \cos(2t + 30^\circ) \quad \omega = 2
\]

\[
H(\omega) = \frac{1}{2+j\omega} = \frac{1}{2+j^2}
\]

\[
|H(\omega)| = \frac{1}{2\sqrt{2}}
\]

\[
\theta(\omega) = \arctan \left( \frac{1}{2} \right) = 45^\circ
\]

Thus \( y(t) = \frac{1}{\sqrt{2}} \cos(2t - 15^\circ) \)
[3]. (30 pts) Consider the RL circuit shown below. 

(a). (15 pts) Write down a differential equation to describe the input-output relationship by using \( v_i(t) \) for input and \( v_o(t) \) for output.

\[
\begin{align*}
\frac{\dot{I}(t)}{\frac{V_o(t)}{1}} &= \frac{V_o(t)}{1} \\
\frac{V_i(t)}{1} &= \frac{dV_o(t)}{dt} + V_o(t) \\
\end{align*}
\]

(1)

(b). (15 pts) Find \( v_o(t) \) for zero initial condition for the inductor and \( v_i(t) = u(t) \). (Hint: Voltage across an inductor is \( L\dot{i}/dt \). The Laplace transformation (LT) method would be handy for this problem. LT\[dz(t)/dt\] = s\(Z(s)\), LT\[K.u(t)\] = \(K/s\), and \( \text{InverseLT}[a/(s+b)] = a \exp(-bt).u(t) \).

\[
\begin{align*}
\mathcal{L}\left[\frac{\dot{V}_o(t)}{1}\right] &= \frac{1}{s} \\
\mathcal{L}\left[\frac{dV_o(t)}{dt}\right] &= sV_o(s) \\
\mathcal{L}\left[V_o(t)\right] &= V_o(s) \\
\end{align*}
\]

Thus Eq(1) in \( S\)-domain is \( \frac{1}{s} = sV_o(s) + V_o(s) \) (2)

\[
\begin{align*}
V_o(s) &= \frac{1}{s(5+1)} \\
&= \frac{1}{s} + \frac{-1}{s+1} \\
V_o(t) &= \mathcal{L}^{-1}V_o(s) = (1 - e^{-t})u(t) \\
\end{align*}
\]
[4]. (20 pts) For \( x(t) \) below, find its Fourier coefficients for \( k=0, 5 \) and \(-5\).

\[
C_k = \frac{1}{T_0} \int_{-T_0}^{T_0} x(t)e^{-jkw_0t} \, dt
\]

Use Fourier coefficients can be found by the following formula:

Where \( T_0 \) stands for one period. Please show all of your work. Applying to the formula for \( C_k \) without showing the method by integration above would not get any credit.

\[
C_0 = \frac{2}{\pi} \quad C_{-5} = \frac{4}{-35\pi} \quad C_5 = \frac{4}{35\pi}
\]

\[
C_0 = \frac{1}{T_0} \int_{-T_0}^{T_0} x(t) \, dt = \frac{1}{\pi} \left[ 4 \right]_0^\pi = \frac{2}{\pi}
\]

\[
C_5 = \frac{1}{T_0} \int_{-T_0}^{T_0} x(t) e^{-j5\omega_0t} \, dt = \frac{1}{2\pi} \left[ \pi e^{-j5\pi} - e^{-j5\pi} \right]_0^\pi = \frac{4}{2\pi} \left( -1 - 1 \right) = \frac{4}{-35\pi}
\]

\[
C_{-5} = (C_5)^* = \frac{4}{-35\pi}
\]