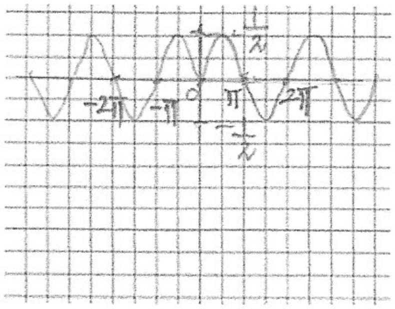
ECE103 Midterm Examination October 31, 2018

Name________________________ ID________________________

[1]. (20 pts) For x(t) = sin(t) u(t), where u(t) = 1 for $t \geq 0$, and 0 for $t<0$.

(a). (8 pts) **Plot its even part $x_e(t)$ in the graph below.**

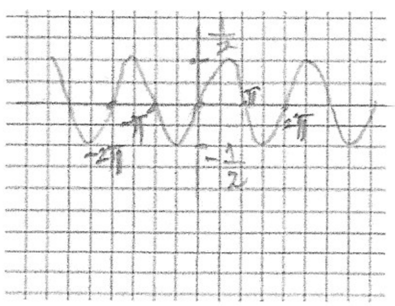
$x_e(t) = \frac{\sin t\, u(t) + \sin(-t)\, u(-t)}{2}$



$x_e(t)$

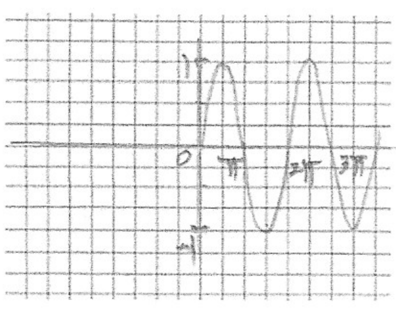
(b). (12 pts) **Plot the even odd part $x_o(t)$ in the top graph below and then show that $x(t)= x_e(t) + x_o(t)$ in the bottom-most graph.**

$x_o(t) = \frac{x(t) - x(-t)}{2}$

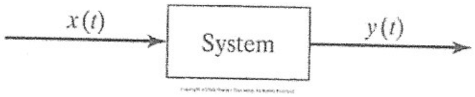


$x_o(t)$

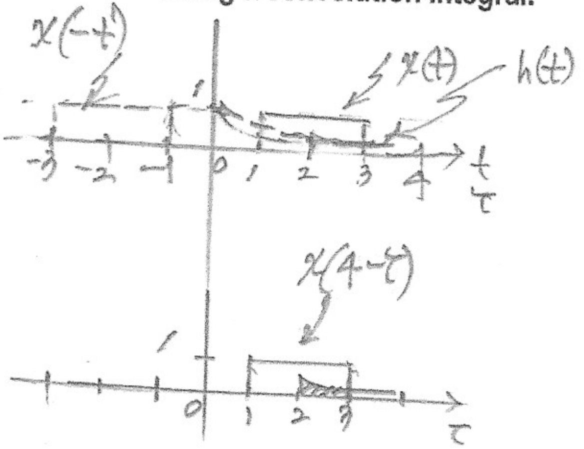
x(t)



[2].(30 pts) Consider a system with its x(t)-to-y(t) relationship characterized by h(t).



(a). (15 pts) When h(t) = exp(-2t) u(t-2) and x(t) = [u(t-1) - u(t-3)], **find y(t) at t=4 by using a convolution integral.**



$$y(t=4) = \int_2^3 h(\tau)\, x(4-\tau)\, d\tau$$

$$= \int_2^3 e^{-2\tau} d\tau = \left[\frac{e^{-2\tau}}{-2}\right]_2^3$$

$$= \frac{e^{-4} - e^{-6}}{2}$$

(b). (15 pts) Let H(ω) = 1/(2+jω), where ω = 2πf, **find y(t) for x(t) = 2cos (2t+30°).** You can express y(t) as a sinusoidal function with a phase angle in the form of arctan (a/b) with specific numerical values of a, b. For reference, arctan x values for x= 0,1,2 are 0, 45 and 63.4 degrees, respectively.

$x(t) = 2\cos(2t + 30^\circ)$ $\omega_0 = 2$

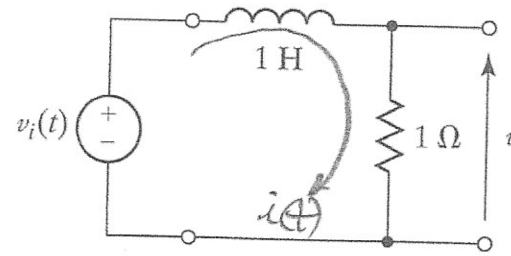
$$H(\omega)\Big|_{\omega=\omega_0} = \frac{1}{2 + j\omega(\omega_0 = 2)} = \frac{1}{2 + j2}$$

$$|H(\omega_0)| = \frac{1}{2\sqrt{2}}$$

$$\angle H(\omega_0) = 0 - \tan^{-1}\frac{2}{2} = -45^\circ$$

Thus $y(t) = \frac{1}{\sqrt{2}} \cos(2t - 15^\circ)$

[3]. (30 pts) Consider the RL circuit shown below.



(a).(15 pts) **Write down a differential equation to describe the input-output relationship by using v_i(t) for input and v_o(t) for output.**

$i(t) = \frac{v_o(t)}{1\Omega} = v_o(t)$

$v_i(t) = 1\,\frac{dv_o(t)}{dt} + v_o(t)$ (1)

(b).(15pts) Find v_o(t) for zero initial condition for the inductor and v_i(t)=u(t). (Hint: Voltage across an inductor is Ldi/dt. The Laplace transformation (LT) method would be handy for this problem. LT[dz(t)/dt]=sZ(s), LT[K.u(t)]=K/s,and InverseLT[a/(s+b)]= a.exp(-bt).u(t)).

$\mathcal{L}[v_i(t) = u(t)] = \frac{1}{s}$

$\mathcal{L}\left[\frac{dv_o(t)}{dt}\right] = sV_o(s)$

$\mathcal{L}[v_o(t)] = V_o(s)$

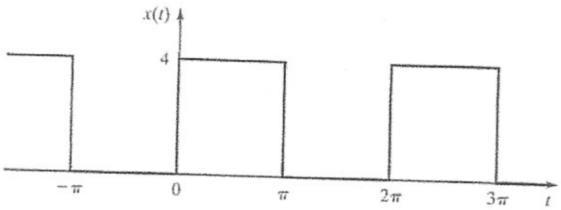
thus Eq(1) in s-domain is $\frac{1}{s} = sV_o(s) + V_o(s)$ (2)

$V_o(s) = \frac{1}{s(s+1)}$ (3)

$= \frac{1}{s} + \frac{-1}{s+1}$

$v_o(t) = \mathcal{L}^{-1} V_o(s) = (1 - e^{-t})\,u(t)$

[4]. (20 pts) For x(t) below, find its Fourier coefficients for k=0, 5 and -5.



Use Fourier coefficients can be found by the following formula:

$$C_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

Where To stands for one period. Please show all of your work. Applying to the formula for Ck without showing the method by integration above would not get any credit.

C_0= ____2____ C_{-5}= $\frac{4}{-j5\pi}$ C_5= $\frac{4}{j5\pi}$

$$C_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = \frac{1}{2\pi}\int_0^{\pi} 4\,dt = \frac{2}{\pi}$$

$$C_5 = \frac{1}{T_0}\int_{T_0} x(t)\,e^{-j5\omega_0 t}\,dt \underset{\uparrow}{=} \frac{1}{2\pi}\int_0^{\pi} 4e^{-j5t}\,dt = \frac{4}{2\pi}\left.\frac{e^{-j5t}}{-j5}\right|_0^{\pi}$$

$\omega_0 = 2\pi/T_0 = 1$

$T_0 = 2\pi$

$$= \frac{4}{2\pi}\left(\frac{e^{-j5\pi}-1}{-j5}\right) \underset{\uparrow}{=} \frac{4}{2\pi}\left(\frac{-1-1}{-j5}\right) = \frac{4}{j5\pi}$$

$e^{-j5\pi} = -1$

$$C_{-5} = (C_5)^* = \frac{4}{-j5\pi}$$