

solution

ECE103 Quiz 3 Oct. 22, 2018

Name _____ Student ID No. _____

Consider a system with $h(t) = e^{-2t} u(t)$.

For its periodic input signal $x(t) = \sin 2t + \cos 8t$, find the Fourier coefficient of the corresponding output signal $y(t)$ for $\omega = 8$. (Hint find $H(j\omega)$, its magnitude and phase angle and apply them to find c_k^y where k corresponds to $k\omega_0 = 8 = \omega$.)

(a). (4 points) Find $H(j\omega)$.

$$\begin{aligned}
 H(j\omega) &= \int_0^{\infty} e^{-2t} e^{-j\omega t} dt = \int_0^{\infty} e^{-(2+j\omega)t} dt \\
 &= \frac{1}{-(2+j\omega)} e^{-(2+j\omega)t} \Big|_0^{\infty} = \frac{1}{2+j\omega} = \frac{1}{\sqrt{4+\omega^2}} e^{-j \tan^{-1} \frac{\omega}{2}}
 \end{aligned}$$

(b). (6 points) Find c_k^y for $k\omega_0 = 8 = \omega$.

$$c_k^y = |H(j\omega)| |c_k^x| e^{j(c_k^x - \tan^{-1} \frac{\omega}{2})} \Big|_{\omega=8}$$

$$= \frac{1}{\sqrt{4^2 + 4^2 \cdot 2^2}} e^{j(6 - \tan^{-1} \frac{8}{2})}$$

$$= \frac{1}{4\sqrt{5}} e^{-\tan^{-1} 4} \stackrel{\omega=8}{\rightarrow} \frac{1}{4\sqrt{5}} \cos(8t - 75.96^\circ)$$