solution

ECE103 Quiz 7, November 19, 2018

Name______________________________ Student ID No.______________________________

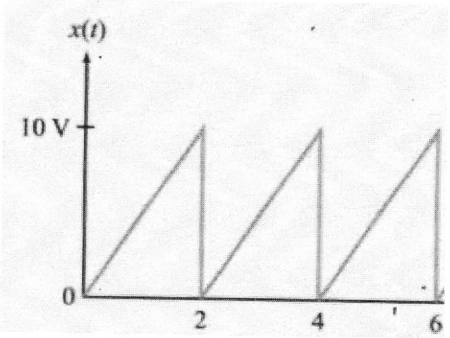
This quiz tests your knowledge on Laplace transform for signal processing.

Part 1 (4 points): For H(s) = 1000 (s+100)/[(s+10) (s+1000)], find h(t) at t = 0_+ . (Hint: you can apply the initial value limit theorem.)

$$h(0_+) = \lim_{s\to\infty} sH(s) = \lim_{s\to\infty} \frac{1000s(s+100)}{(s+10)(s+1000)} = \lim_{s\to\infty} \frac{1000(1+\frac{100}{s^2})}{(1+\frac{10}{s})(1+\frac{1000}{s})}$$

$$= \frac{1000(1+0)}{(1+0)(1+0)} = \underline{1000}$$

Part 2 (6 points) Find F(S), Laplace Transform of the function f(t)= x(t) x rect ((t-3) l 6) of which x(t) is shown below: (Hint: rect (t l T) = 1 for ltl < T/2. 0 elsewhere.)



$$f(t) = \begin{cases} x(t) & \text{for } 0<t<6 \\ 0 & \text{elsewhere} \end{cases}$$

$$\mathcal{L}[f(t)] = \mathcal{L}\int_{0_-}^{6} x(t)e^{-st}\,dt$$

$$= \left[\int_{0_-}^{2} 5t\,e^{-st}\,dt\right]^{**} + \int_{2}^{4} 5(t-2)e^{-st}\,dt + \int_{4}^{6} 5(t-4)e^{-st}\,dt\,.$$

For **

$$\int_{0_-}^{2} 5t\,d\left(\frac{e^{-st}}{-s}\right) = 5t\,\frac{e^{-st}}{-s}\Big|_{0_-}^{2} - 5\int_{0_-}^{2}\frac{e^{-st}}{-s}dt = 10\,\frac{e^{-2s}}{-s} - 0 - 5\left[\frac{e^{-st}}{+s^2}\right]_{0_-}^{2}$$

$$= 10\frac{e^{-2s}}{-s} - 5\left[\frac{e^{-2s}}{s^2} - \frac{1}{s^2}\right] = \frac{5}{s^2}\left[1 - e^{-2s}(1+2s)\right]$$

$$\Rightarrow F(s) = \underline{\frac{5}{s^2}\left(1-e^{-2s}(1+2s)\right)}\,\underline{\left[1 + e^{-2s} + e^{-4s}\right]}$$

delay by 2 delay by 4