Solution

ECE103 Quiz 2 October 15, 2018

Name_________________________ Student ID No._________________________

This quiz is to find outputs \( y(t) \) of systems characterized by their impulse responses \( h(t) \) for given inputs \( x(t) \) by convolution integral, i.e.,
\[ y(t) = x(t) * h(t). \]

Consider the following system:

\[ x(t) \rightarrow h_1(t) \rightarrow h_2(t) \rightarrow y(t) \]

Here \( h_1(t) \) is “integration” over time domain, \( h_2(t) \) is “time delay” by 1 second.

1. (4 points) Express \( y(t) = h(t) \) in mathematical form for \( x(t) = \delta(t) \), impulse. (hint: \( h(t) \) can be expressed by a basic function such as \( \delta(t) \), \( u(t) \), \( R(t) \), \( \exp(t) \), etc. or its modification.)

\[ h(t) = u(t-1) \]

2. (6 points) Find \( y(t) \) for \( x(t) = R(t) \) by convolution \( x(t)*h(t) \), where \( R(t) = 0 \) for \( t < 0 \) and \( t \) for \( t > 0 \).

Show all work, both graph and mathematical procedures.

\[ \frac{1}{2} (t-1)^2 \]

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\[ \delta(t) = \begin{cases} \frac{1}{\tau}, & \text{for } t > 0 \\ 0, & \text{for } t < 0 \end{cases} \]

\[ \text{checking} \]

\[ h_1(x(t)) = \int_x^t \delta(t) \, dt \]

\[ = \frac{1}{\tau} (t-1)^2 u(t-1) \]

\[ h_2[h_1(x(t))] = \frac{1}{\tau} (t-1)^2 u(t-1) \]