ECE103_F18_HW#3 Oct 15, 2018

On Fourier Series

[1].

4.1. For the harmonic series

$$f(t) = \cos 2t + 3\cos 4t,$$

show that the Fourier coefficients of the exponential form of $f(t)$ are given by the following:

(a) $C_0 = 0$
(b) $C_1 = 0.5$
(c) $C_2 = 1.5$
(d) $C_k = 0, k \geq 3$

[2].

4.4. A periodic signal $x(t)$ is expressed as an exponential Fourier series:

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_o t}.$$

Show that the Fourier series for $\hat{x}(t) = x(t - t_o)$ is given by

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} \hat{C}_k e^{jk\omega_o t},$$

in which

$$|\hat{C}_k| = |C_k| \quad \text{and} \quad \angle\hat{C}_k = \angle C_k - k\omega_o t_o.$$

[3].

4.6. This problem will help illustrate the orthogonality of exponentials. Calculate the following integrals:

(a) $\int_0^{2\pi} sin^2(t)dt$

(b) $\int_0^{2\pi} sin^2(2t)dt$

(c) $\int_0^{2\pi} sin(t)sin(2t)dt$

(d) Explain how the results of parts (a), (b), and (c) illustrate the orthogonality of exponentials.

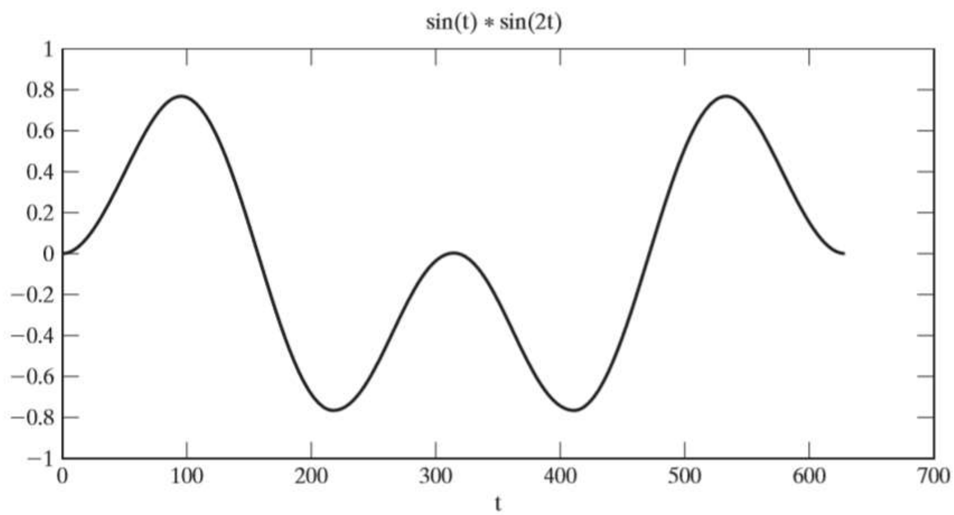

Figure P4.6

[4].

4.17. Consider the signals in Figure P4.11. For k sufficiently large, the Fourier coefficient of the k^{th} harmonic decreases in magnitude at the rate of $1/k^m$. Use the properties in Section 4.4 to find m for the signals shown in the following figures:

(a) Figure P4.11(a)
(b) Figure P4.11(b)
(c) Figure P4.11(c)
(d) Figure P4.11(d)

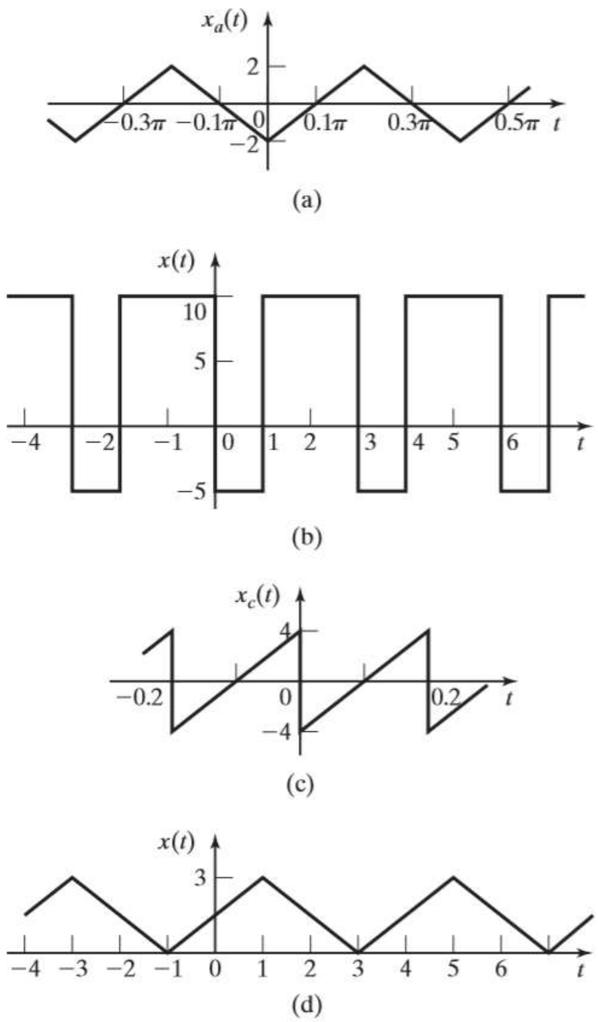

Fig. P4.11

[5].

4.26. Consider the *RC* circuit of Figure P4.26:

(a) The square wave of Table 4.3 is applied to the input of this circuit, with $T_0 = 2\pi$ s and $X_0 = 10$ V. Solve for the frequency spectrum of the output signal. Give numerical values for the amplitudes and phases of the first three nonzero sinusoidal harmonics.

(b) Verify the results in part (a), using MATLAB.

(c) Let the input of the circuit be as in part (a), but with a dc value of 20 V added to the square wave. Solve for the frequency spectrum of the output signal. Give numerical values for the dc component and first three nonzero sinusoidal harmonics.

(d) Is the circuit low pass? Why?

(e) The period of the square wave is changed to $T_0 = \pi$. State the effects of this change on the answers to parts (a) and (c), without solving these parts again. Give the reasons for your answers.

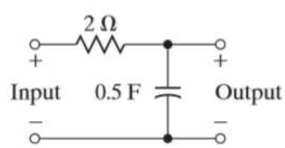

Figure P4.26

[6].

4.30. Consider the system of Figure P4.30, with $h(t) = e^{-\alpha t}u(t)$.

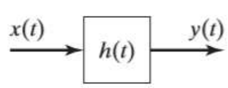

Figure P4.30

(a) For what values of α will the system be BIBO stable?

(b) Assume that the system is BIBO stable. The input signal is $x(t) = \sin t + \cos 3t$. Find $y(t)$.

4.31. Consider the system of P4.30, with $h(t) = \alpha e^{-\alpha t}u(t)$, $\alpha > 0$. This is a *low-pass filter*.

(a) The input signal is $x(t) = \sin^2 2t$. Find $y(t)$. Notice that the higher frequency components are *attenuated* more than the dc component.

(b) Repeat part (a) with $x(t) = 1 + \cos t + \cos 8t$. Again, notice how the higher frequency component ($\cos 8t$) is attenuated more than the lower frequency components ($\cos t$ and the dc term).