ECE103 HW#9 November 26, 2018

Sampling Theorem

- Let $x(t)$ be a real-valued, continuous-time, lowpass signal *bandlimited* to a maximum frequency of B Hz.
- Let $x[n] = x(nT_s)$ be the sequence of numbers obtained by *sampling* $x(t)$ at a sampling rate of f_s samples per second, that is, every $T_s = 1/f_s$ seconds.
- Then $x(t)$ can be *uniquely* reconstructed from its samples $x[n]$ if and only if $f_s > 2B$. The sampling rate must exceed double the bandwidth.
- For a bandpass signal of bandwidth B, a modified constraint applies, as discussed later in Section 6-13.3.

The minimum sampling rate $2B$ is called the ***Nyquist sampling rate***. Although the actual units are $2B$ samples per second, this is usually abbreviated to $2B$ "Hertz," which has the same dimensions as $2B$ samples per second.

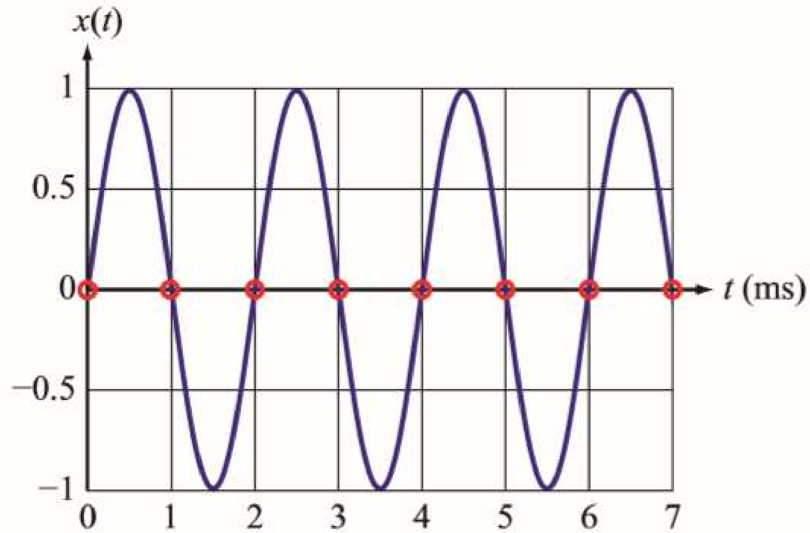

Figure 6-71: A 500 Hz sinusoid, $x(t) = \sin(1000\pi t)$, sampled at the Nyquist rate of 1 kHz.

Example 6-18: Do We Need $f_s > 2B$ or $f_s \geq 2B$?

A 500 Hz waveform given by $x(t) = \sin(1000\pi t)$ is sampled at the Nyquist sampling rate of 1 kHz (1000 samples per second). Obtain an expression for the sampled signal $x_s[n]$ and determine if the original signal can be reconstructed.

Solution: The sine wave's frequency is $f = 500$ Hz, so a sampling rate of 1 kHz is exactly equal to the Nyquist rate. **Figure 6-71** displays the waveform of $x(t)$ and the time locations when sampled at 1 kHz. Every single sample bears a value of zero, because the sampling interval places all sample locations at zero-crossings. Hence,

$$x_s[n] = 0.$$

This is an example of when sampling at exactly the Nyquist rate fails to capture the information in the signal.

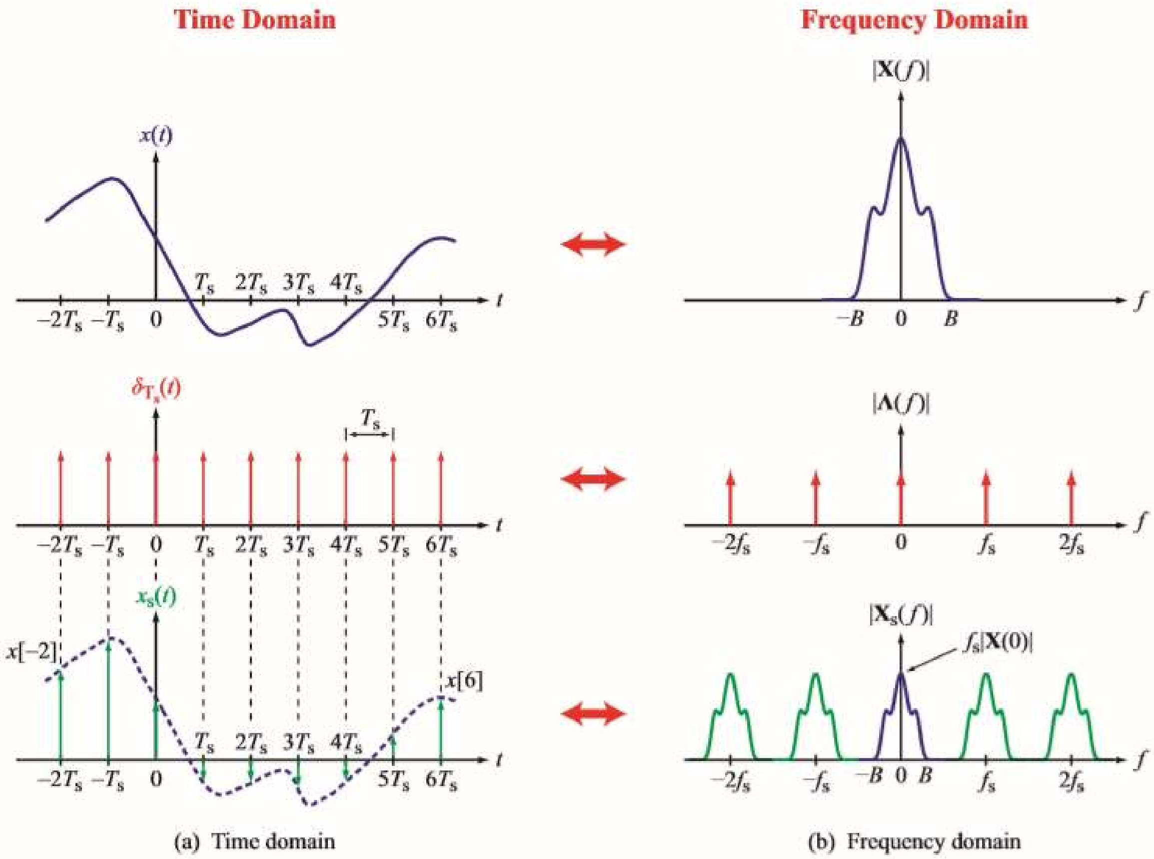

Figure 6-72: In the frequency domain, the spectrum of the sampled signal is a periodic replica of the spectrum of $x(t)$, separated by a spacing $f_S = 1/T_S$.

Section 6-13: Sampling Theorem

*6.62 The spectrum of the trumpet signal for note G (784 Hz) is negligible above its ninth harmonic. What is the Nyquist sampling rate required for reconstructing the trumpet signal from its samples?

6.63 Compute a Nyquist sampling rate for reconstructing signal

$$x(t) = \frac{\sin(40\pi t)\sin(60\pi t)}{\pi^2 t^2}$$

from its samples.

6.64 Signal

$$x(t) = \frac{\sin(2\pi t)}{\pi t}\,[1 + 2\cos(4\pi t)]$$

is sampled every 1/6 second. Sketch the spectrum of the sampled signal.

6.59 FM stereo signals are formed using the system shown in **Fig. P6.59**, where $L(t)$ is the left speaker signal and $R(t)$ is the right speaker signal. Assume both signals are bandlimited to 15 kHz. Also, signal $C(t)$ is a 38 kHz sinusoidal carrier given by $C(t) = 2\cos(76000\pi t)$. Sketch the spectrum of $z(t)$.

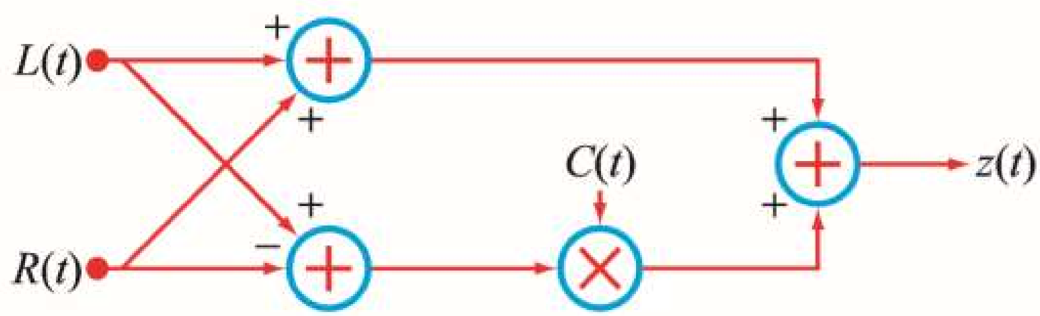

Figure P6.59: FM stereo system (Problem 6.59).

6.67 A signal $x(t)$ has the bandlimited spectrum shown in **Fig. P6.67**. If $x(t)$ is sampled at 10 samples/s and then passed through an ideal brick-wall lowpass filter with a cutoff frequency of 5 Hz, sketch the spectrum of the output signal.

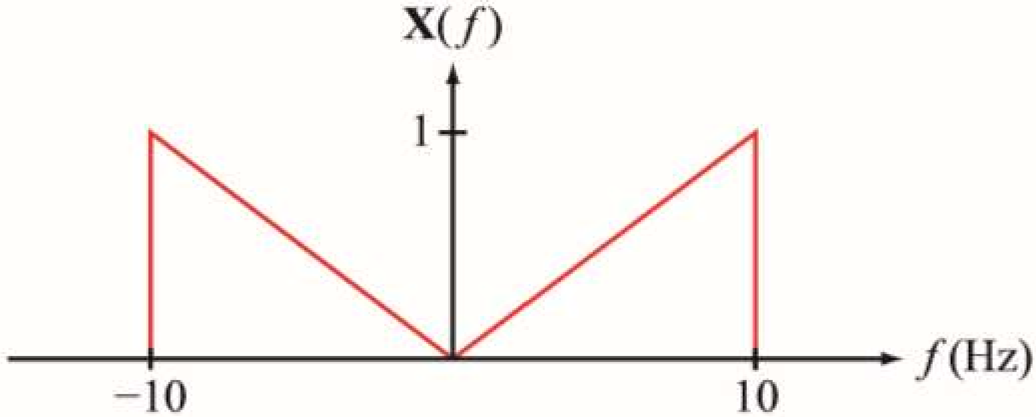

Figure P6.67: Spectrum of $x(t)$ (Problem 6.67).

6.68 A signal $x(t)$, bandlimited to 10 Hz, is sampled at 12 samples/s. what portion of its spectrum can still be recovered from its samples?

6.32 The current $i(t)$ through an inductor can be determined from the voltage $v(t)$ across the inductor using the relation $i(t) = \frac{1}{L} \int_{-\infty}^{t} v(\tau)\, d\tau$, which is equivalent to $v(t) = L\, di/dt$. This requires implementation of the pure integrator system $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$. However, this system is not BIBO-stable, because a (bounded) input $x(t) = u(t)$ leads to an (unbounded) output $y(t) = r(t)$. Using $\mathbf{H}(\omega) = 1/(j\omega + \epsilon)$ is a useful substitute, since $\mathbf{H}(\omega)$ resembles an integrator at high frequencies, but it is also bounded at low frequencies:

$$\mathbf{H}(\omega) \approx \begin{cases} 1/(j\omega) & \omega \gg \epsilon, \\ 1/\epsilon & \omega \ll \epsilon. \end{cases}$$

(a) Compute the transfer function $\mathbf{H}(\mathbf{s})$.

(b) Compute the impulse response $h(t)$.

(c) Compute an LCCDE implementing $\mathbf{H}(\omega)$.

(d) Compute the response to the (bounded) input $x(t) = u(t)$. Is the response bounded?

***4.19** The current source shown in the circuit of **Fig. P4.19** is given by the displayed waveform. Determine $\upsilon_{out}(t)$ for $t \geq 0$ given that $R_1 = 1\ \Omega$, $R_2 = 0.5\ \Omega$, and $L = 0.5$ H.

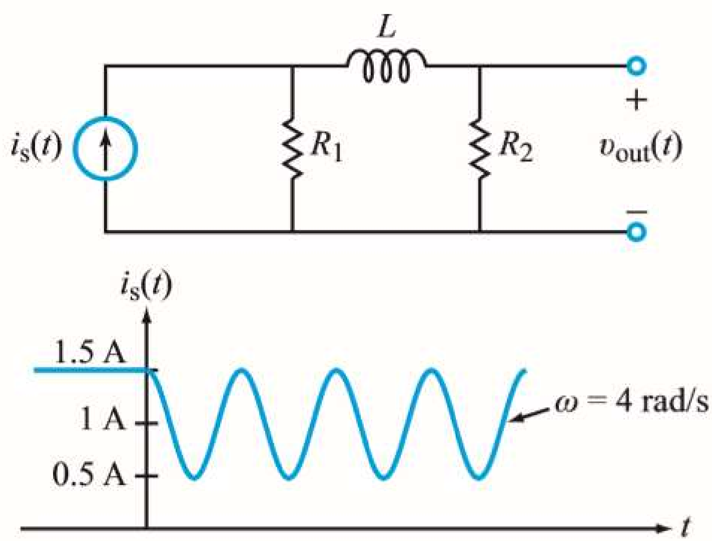

Figure P4.19: Circuit for Problem 4.19 and current waveform for Problems 4.19 and 4.20.

4.20 If the circuit shown in **Fig. P4.20** is excited by the current waveform $i_s(t)$ shown in **Fig. P4.19**, determine $i(t)$ for $t \geq 0$ given that $R_1 = 10\ \Omega$, $R_2 = 5\ \Omega$, and $C = 0.02$ F.

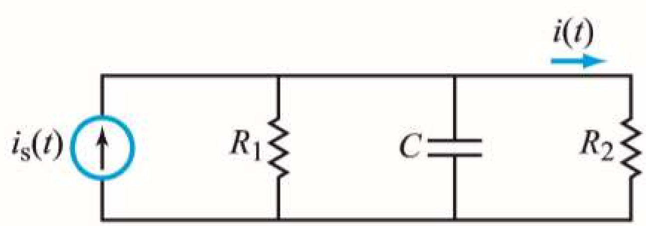

Figure P4.20: Circuit for Problems 4.20 to 4.22.

4.33 In the circuit shown in **Fig. P4.32**,

$$\upsilon_i(t) = 10u(t) \text{ mV},$$

$V_{CC} = 10$ V for both op amps, and the two capacitors had no charge prior to $t = 0$. Analyze the circuit and plot $\upsilon_{out_1}(t)$ and $\upsilon_{out_2}(t)$ using the Laplace transform technique.

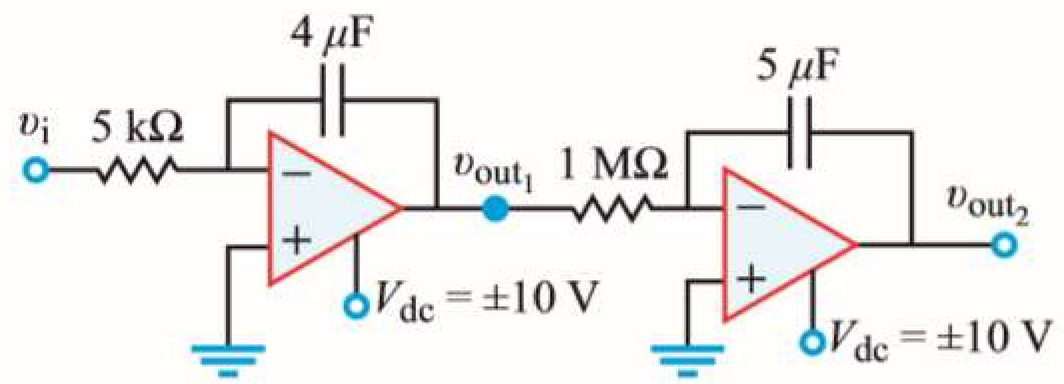

Figure P4.33: Circuit for Problem 4.33 and 4.34.